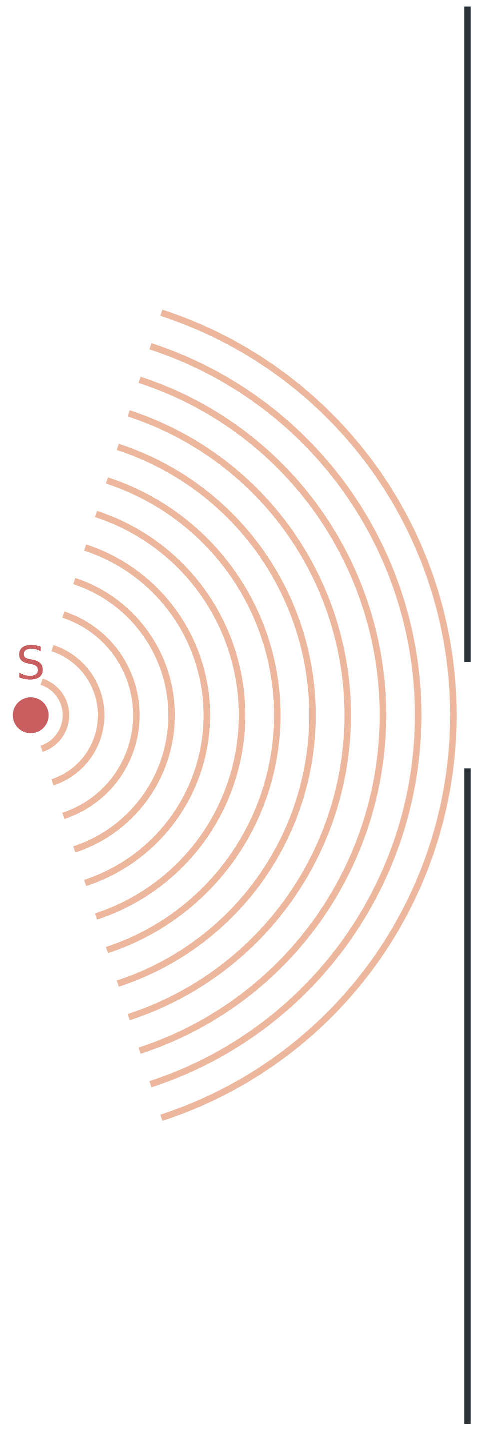
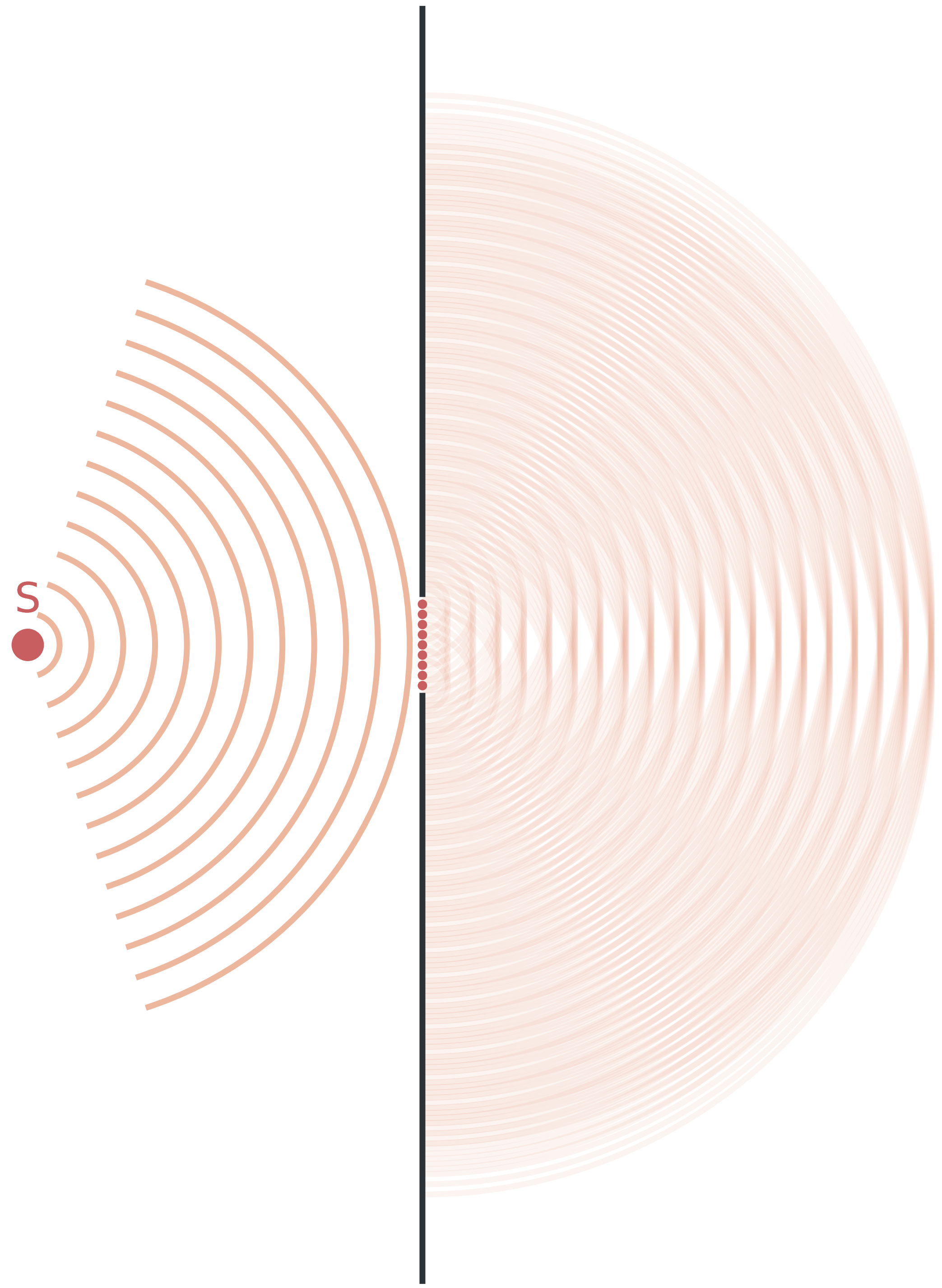
Hola

hablemos de difracción


S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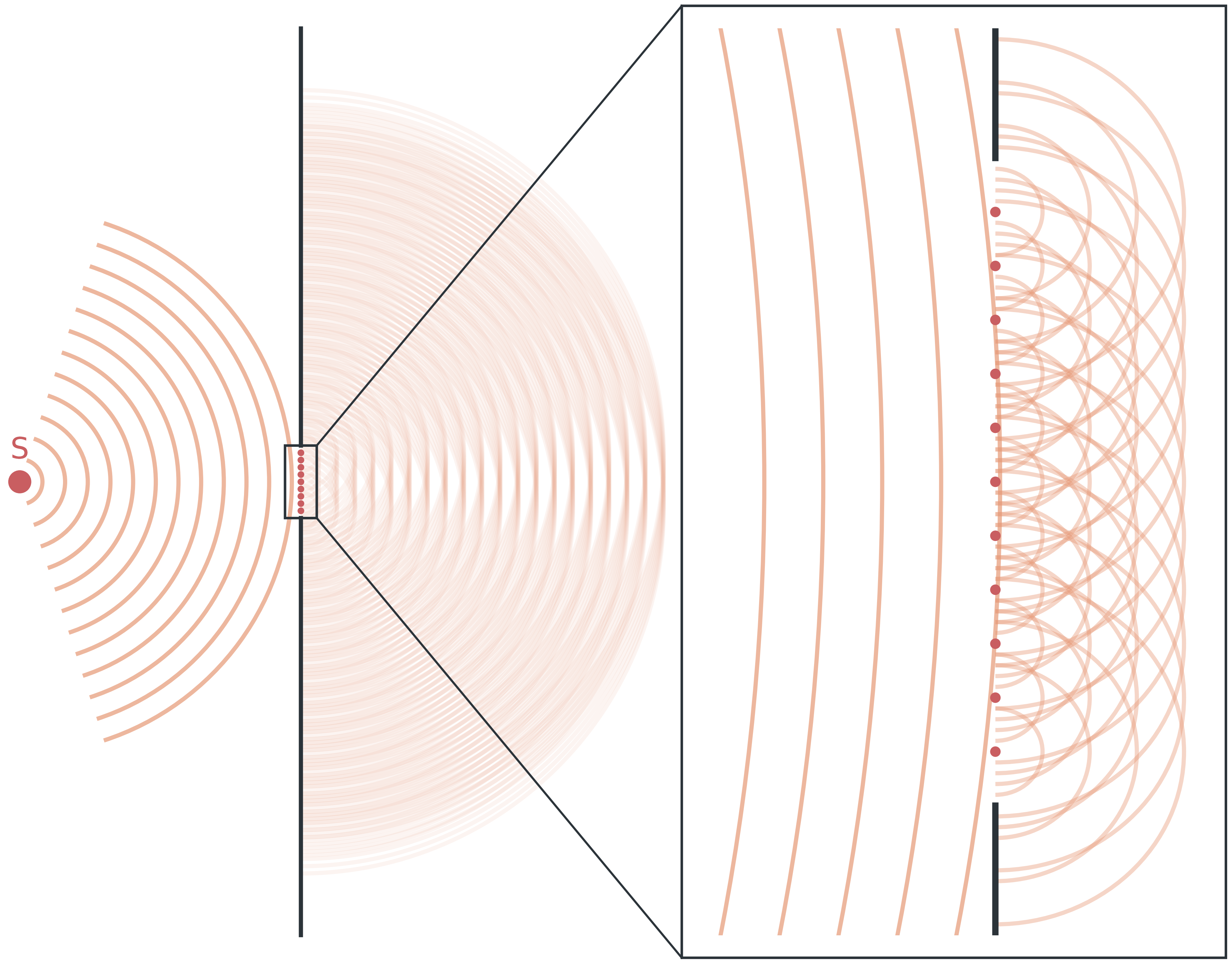
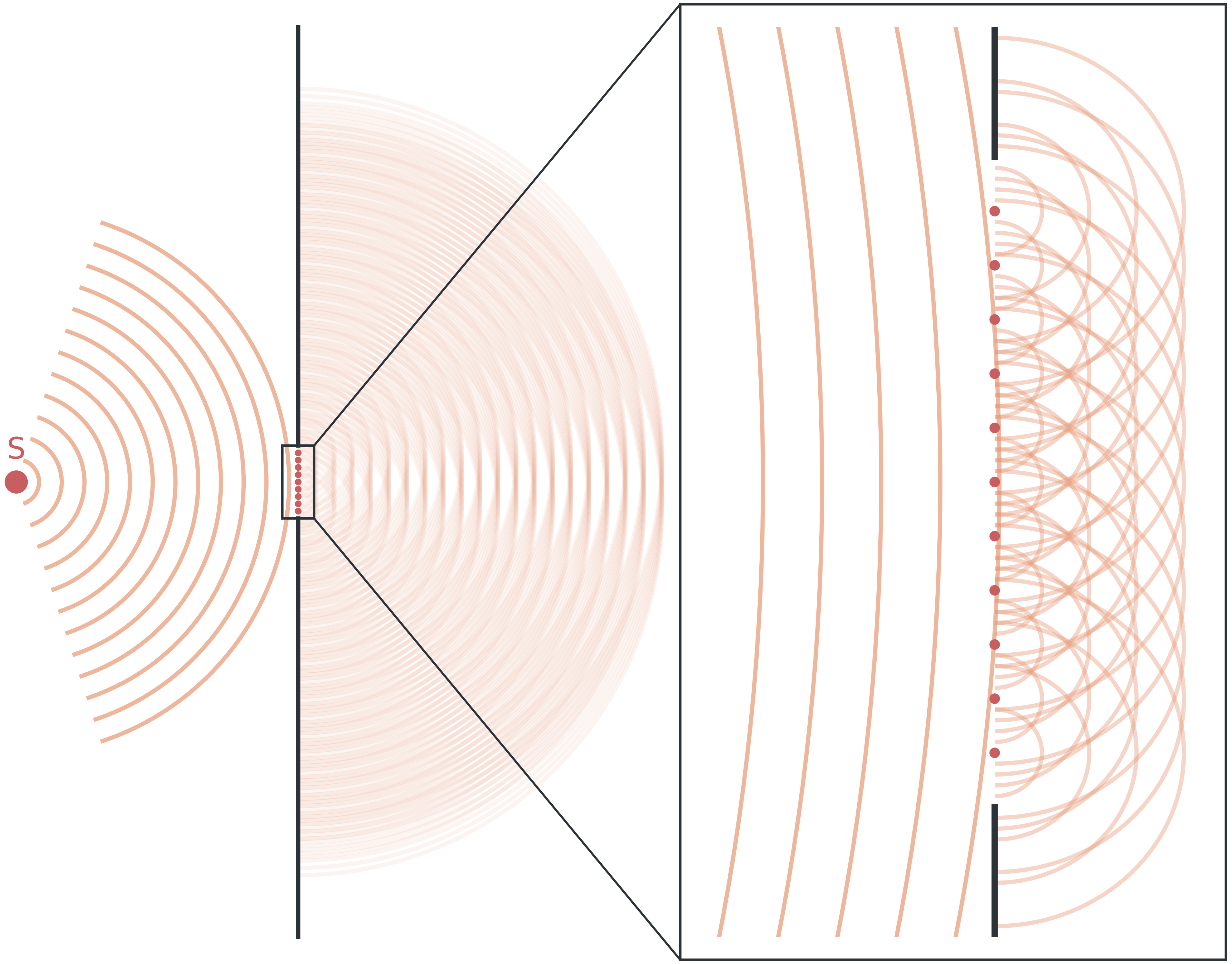
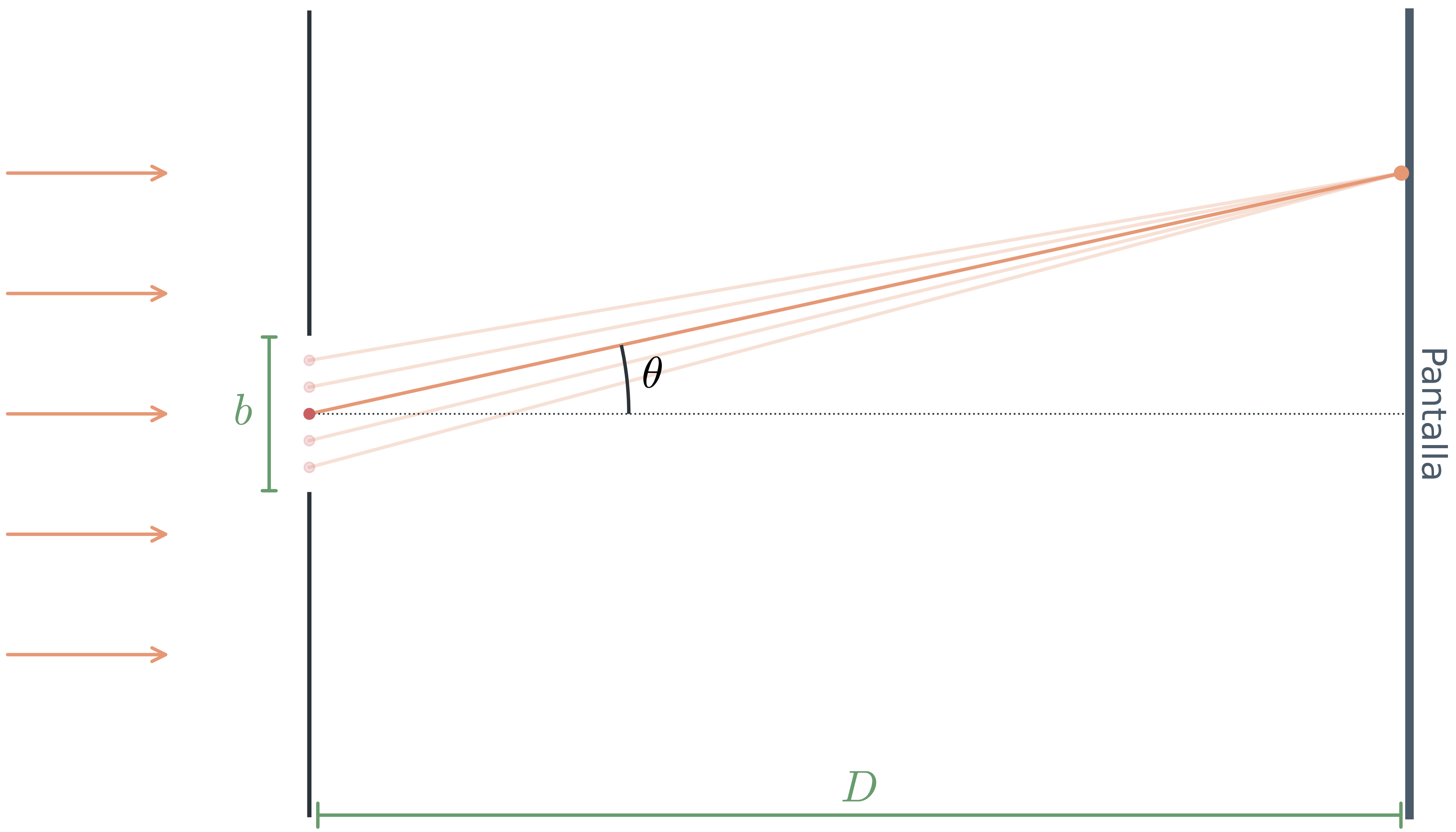

S

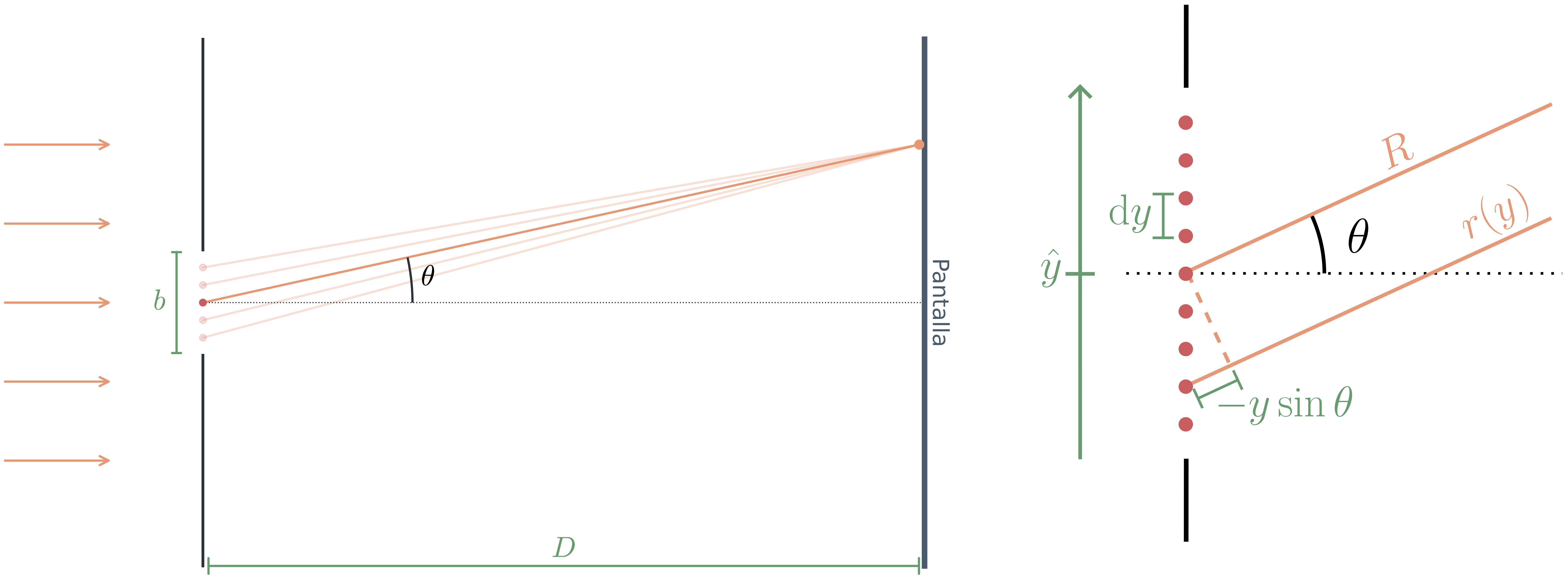


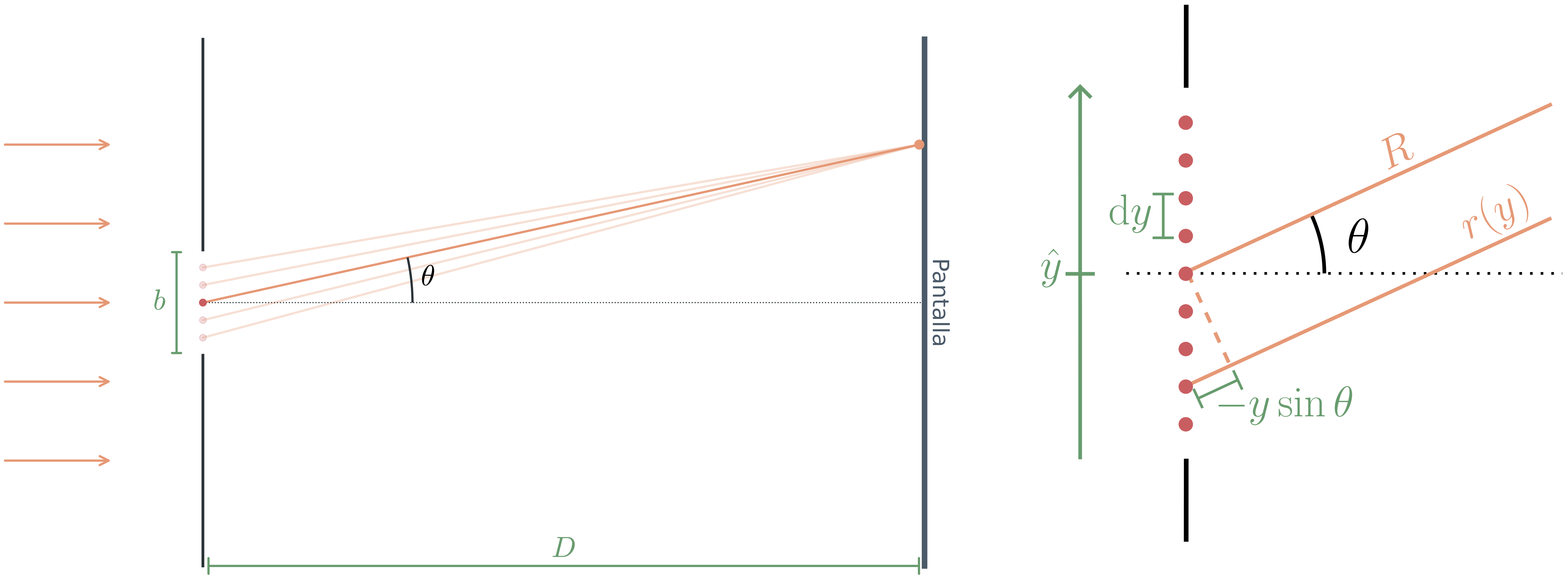
$$\mathbf{E} = \int_{-\infty}^{\infty} \mathrm{d}\mathbf{E}$$

$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{|r(y)|}\, e^{i[\mathbf{k}\cdot\mathbf{r}(y) - \omega\, t]}$$

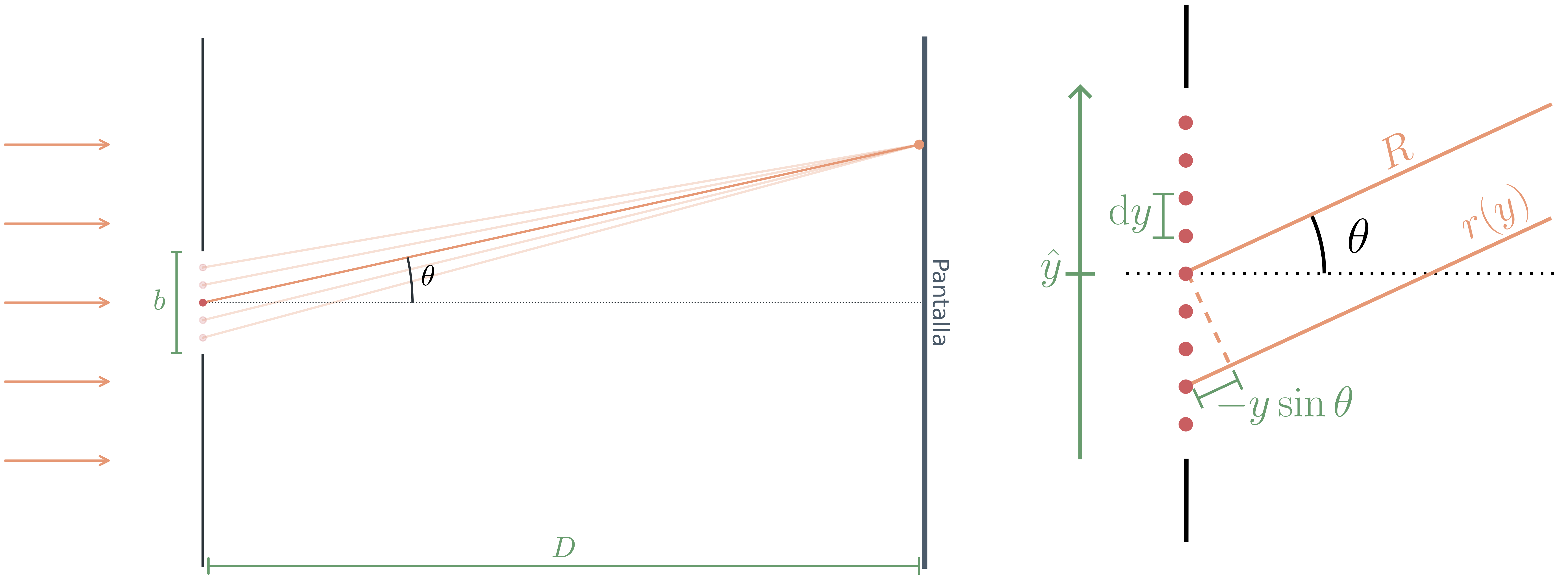
(donde está la rendija)


b
θ
Pantalla
D

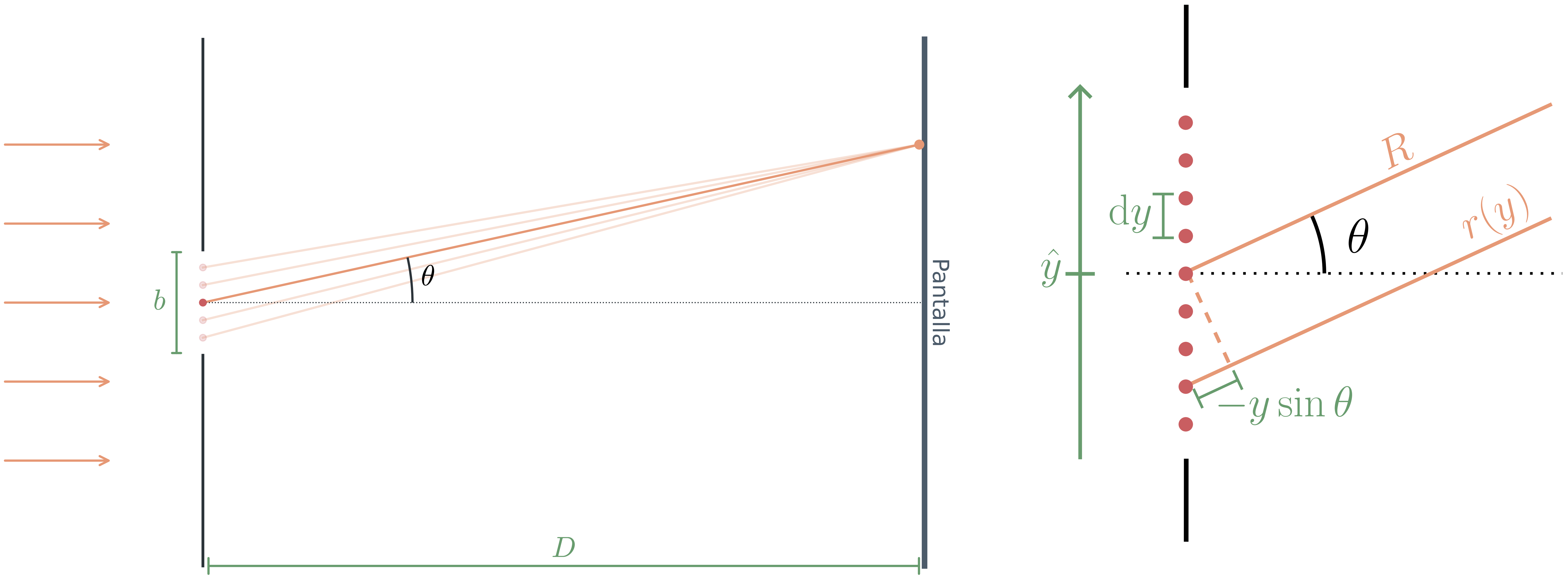

b
θ
D
Pantalla
ŷ
dy
R
r(y)
θ
−y sin θ



$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{|r(y)|}\, e^{i[\mathbf{k}\cdot\mathbf{r}(y) - \omega\, t]} \quad \text{con} \quad r(y) \simeq R - y\, \sin\, \theta \quad \text{y} \quad \frac{1}{|r(y)|} \simeq \frac{1}{R}$$



$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{R}\, e^{i[k(R-y\sin\theta)-\omega\, t]}, \quad \mathbf{E} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \mathrm{d}\mathbf{E}$$



$$\mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{R}\, e^{i[k(R - y\sin\theta) - \omega\, t]}, \quad \mathbf{E} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \mathrm{d}\mathbf{E} = \frac{\mathbf{E_0}}{R}\, e^{i(k\,R - \omega\, t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\, \mathrm{d}y$$

Un comentario antes de continuar...

$$\sin\theta$$

Un comentario antes de continuar...

$$\sin\theta \simeq \theta$$

Un comentario antes de continuar...

$$\sin\theta \simeq \theta \simeq \tan\theta$$

Un comentario antes de continuar...

$$\sin\theta \simeq \theta \simeq \tan\theta \simeq y/D$$

$$\uparrow$$

a veces

$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta} \, \mathrm{d}y$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\, \mathrm{d}y = \frac{\mathbf{E_0}}{R}\, e^{i(kR-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-\ ik\ \ \sin\theta}$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{R}\, e^{i(kR-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{R}\, e^{i(kR-\omega t)}\, \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b = \frac{\mathbf{E_0}}{R}\, e^{i(kR-\omega t)}\, b\, \frac{\sin\left(k\frac{b}{2}\sin\theta\right)}{k\frac{b}{2}\sin\theta}$$

$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\,\mathrm{d}y = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta}\, b = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)}\, b\, \frac{\sin\left(k\frac{b}{2}\sin\theta\right)}{k\frac{b}{2}\sin\theta}$$

$$\Longrightarrow \mathbf{E} = \frac{\mathbf{E_0}}{R}\, b\, e^{i(kR-\omega t)} \frac{\sin\beta}{\beta}, \quad \beta = \frac{bk}{2}\sin\theta$$

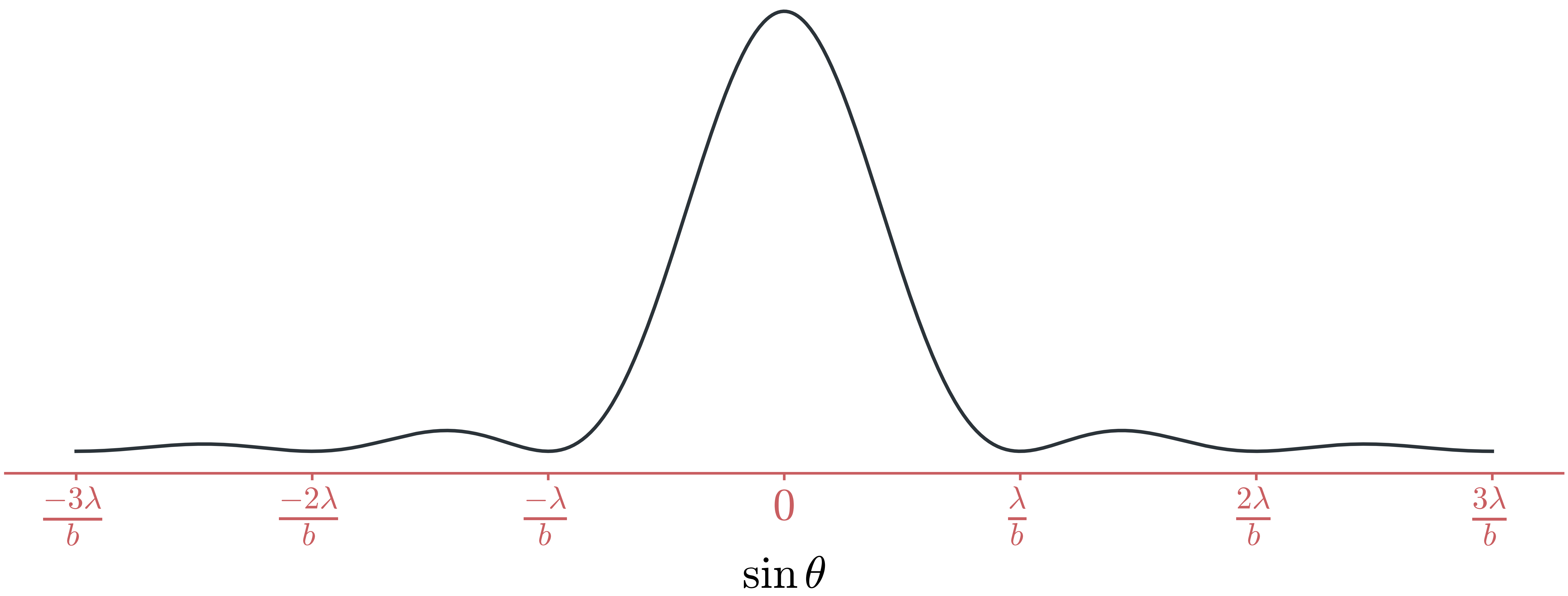
$$\mathbf{E} = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{-iky\sin\theta}\, \mathrm{d}y = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \left. \frac{e^{-iky\sin\theta}}{-ik\sin\theta} \right|_{-\frac{b}{2}}^{\frac{b}{2}}$$

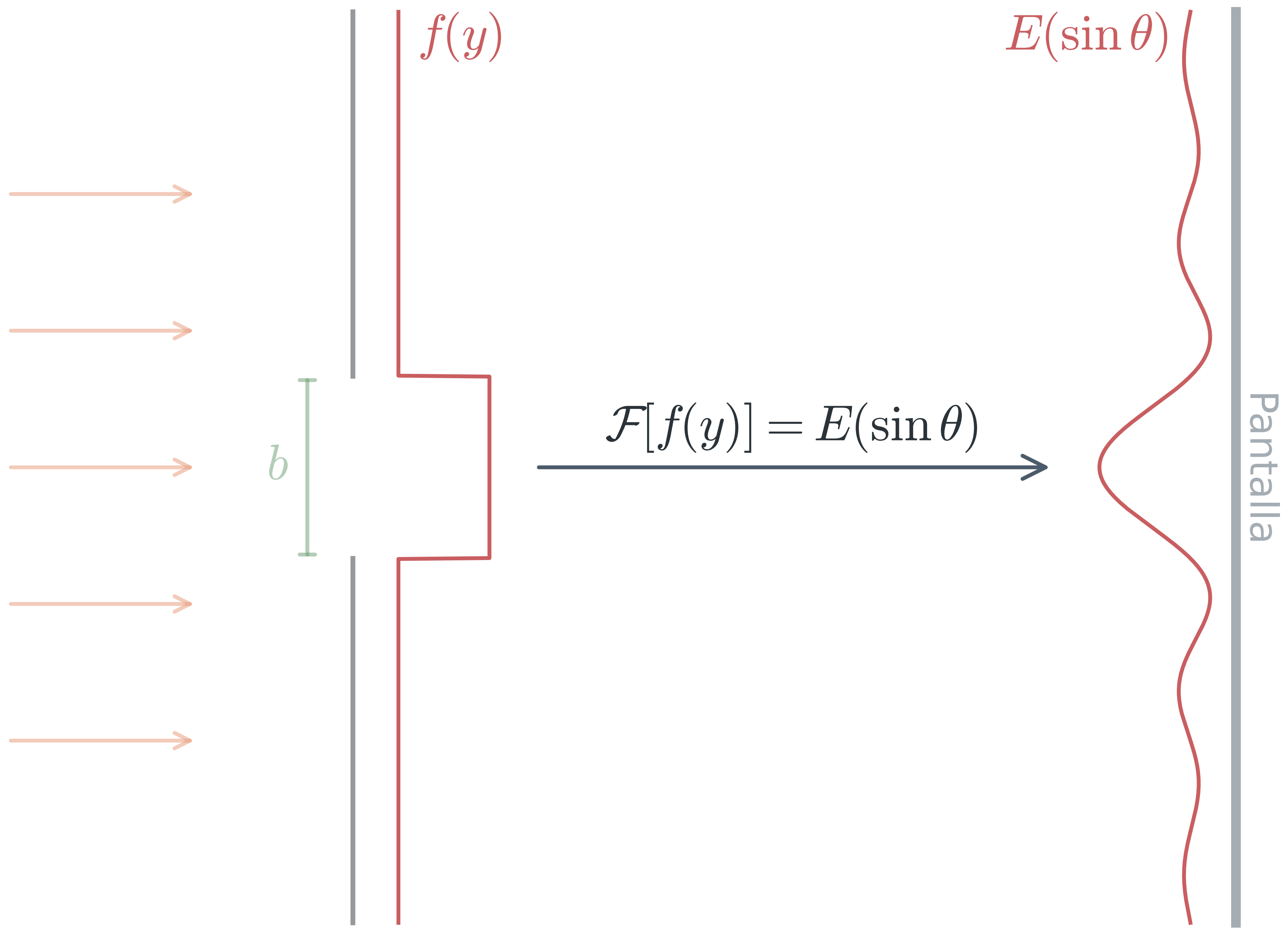
$$= \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} \frac{e^{-ik\frac{b}{2}\sin\theta} - e^{ik\frac{b}{2}\sin\theta}}{-2ik\frac{b}{2}\sin\theta} b = \frac{\mathbf{E_0}}{R} e^{i(kR-\omega t)} b \frac{\sin\left(k\frac{b}{2}\sin\theta\right)}{k\frac{b}{2}\sin\theta}$$

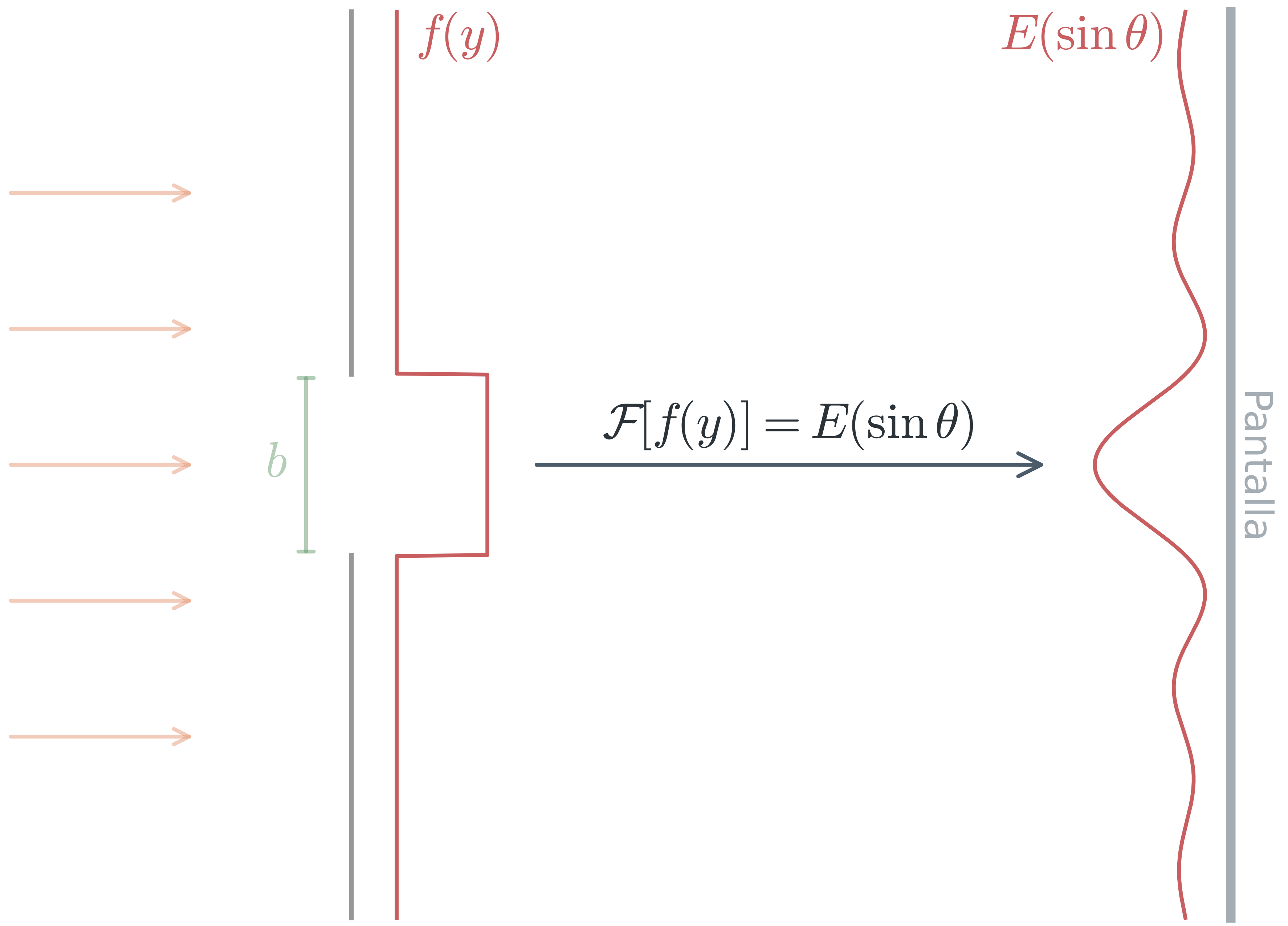
$$\Longrightarrow \mathbf{E} = \frac{\mathbf{E_0}}{R} b\, e^{i(kR-\omega t)} \frac{\sin\beta}{\beta}, \quad \beta = \frac{bk}{2}\sin\theta$$

$$\Longrightarrow I \propto \frac{\mathbf{E}\cdot\mathbf{E}^*}{2} = I_0 \frac{\sin^2\beta}{\beta^2}, \quad I_0 = \frac{1}{2}\left(\frac{bE_0}{R}\right)^2$$

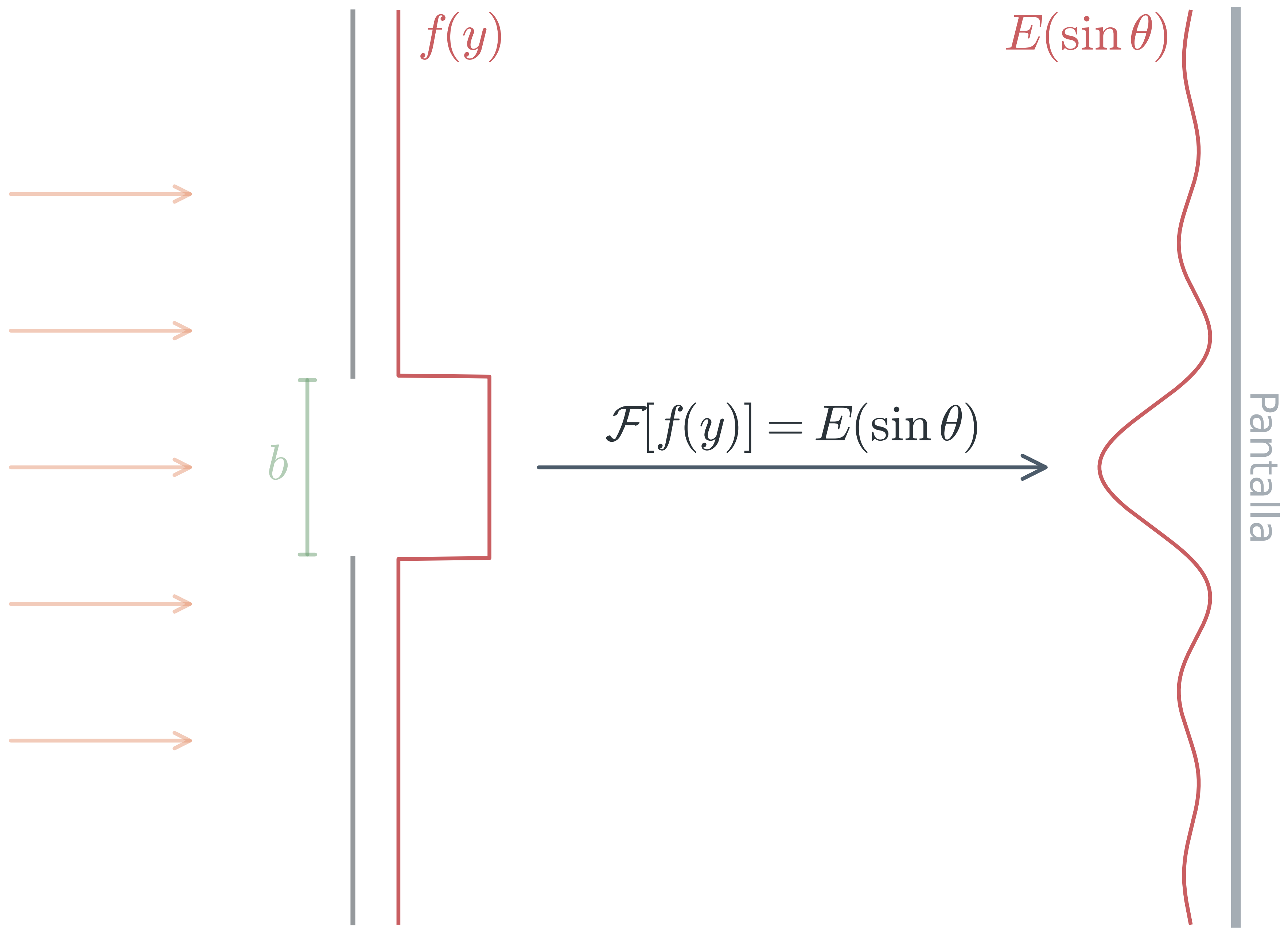
$$I = I_0 \ \text{sinc}^2\left(\frac{bk}{2}\sin\theta\right) = I_0 \ \text{sinc}^2\left(\frac{\pi\, b}{\lambda}\sin\theta\right)$$




$f(y)$
$E(\sin\theta)$
b
$\mathcal{F}[f(y)] = E(\sin\theta)$
Pantalla

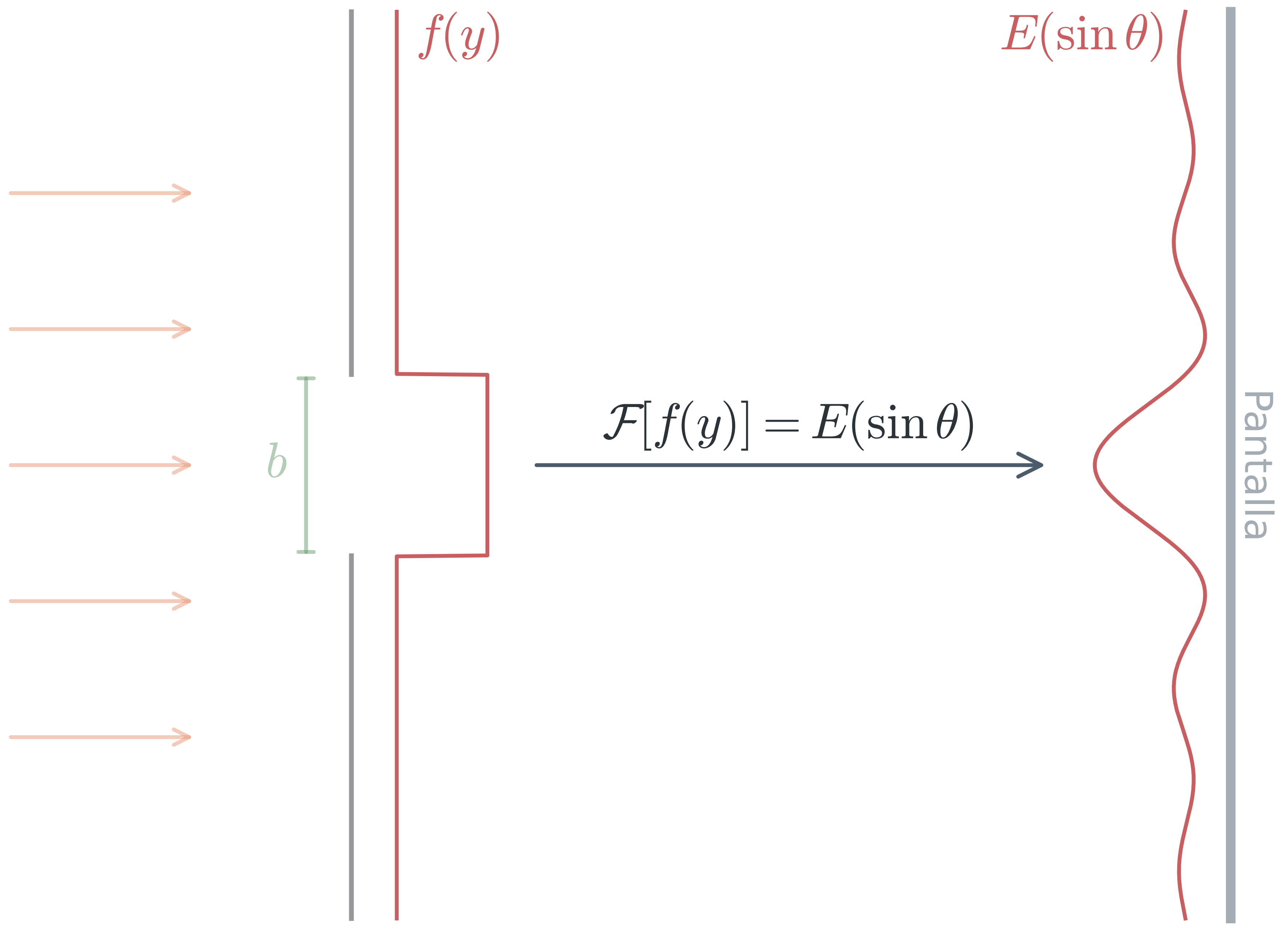


$$f(y) = \begin{cases} E_0 & \text{si } -\frac{b}{2} < y < \frac{b}{2} \\ 0 & \text{si no} \end{cases}$$



$$f(y) = \begin{cases} E_0 & \text{si } -\frac{b}{2} < y < \frac{b}{2} \\ 0 & \text{si no} \end{cases}$$

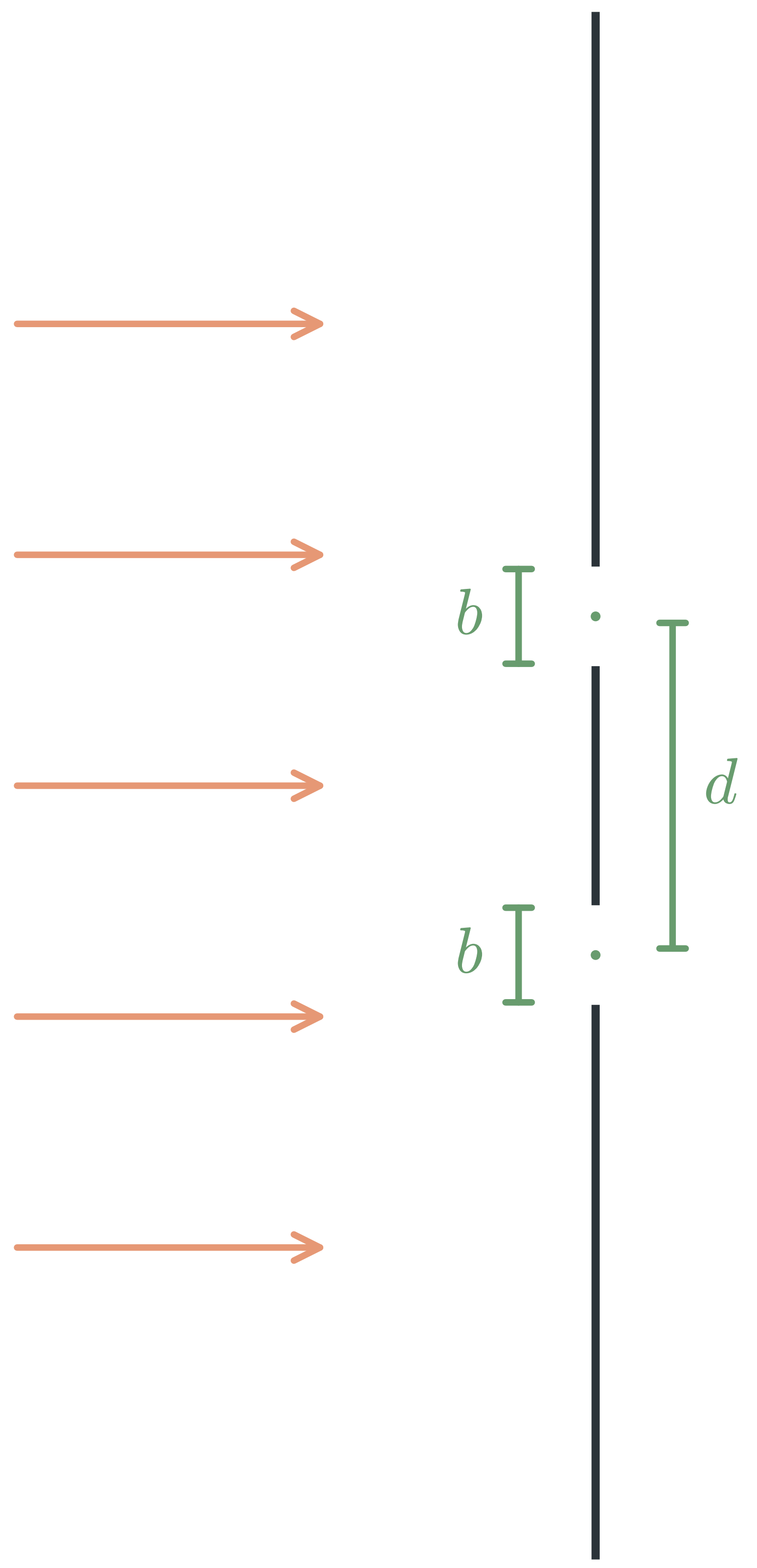
$$E = E_0' \operatorname{sinc}\left(\frac{kb}{2}\sin\theta\right)$$



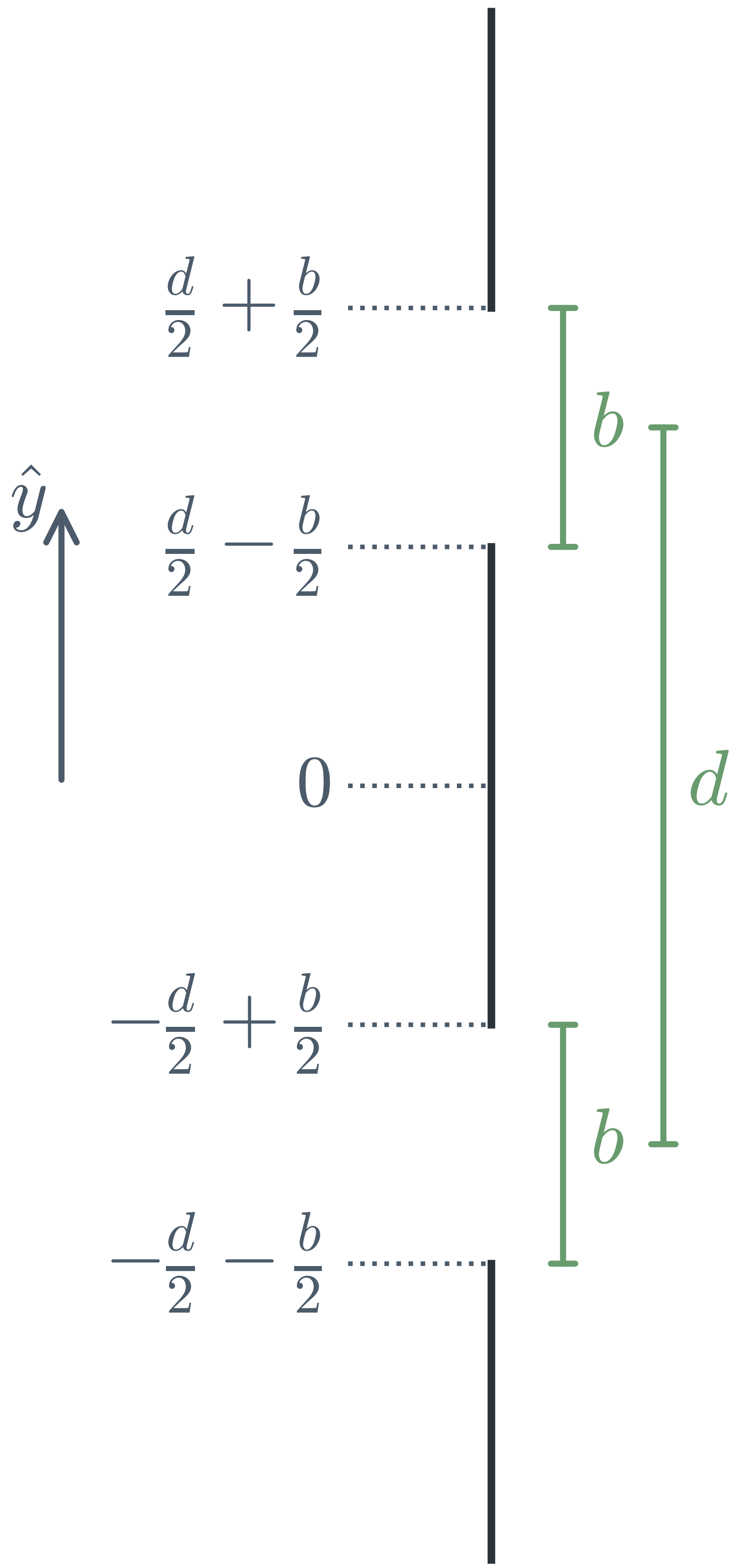
$$f(y) = \begin{cases} E_0 & \text{si } -\dfrac{b}{2} < y < \dfrac{b}{2} \\ 0 & \text{si no} \end{cases}$$

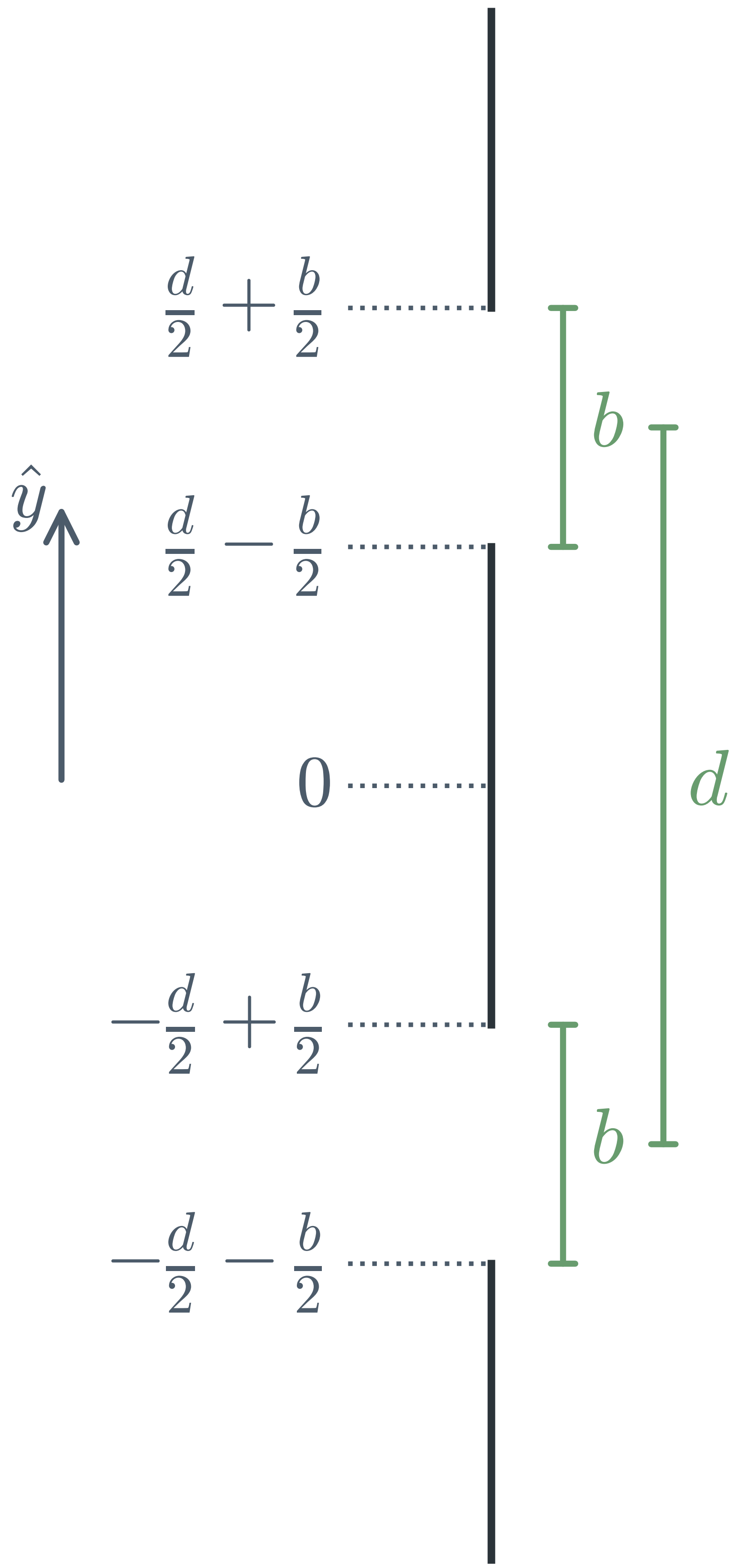
$$E = E_0' \operatorname{sinc}\left(\frac{kb}{2}\sin\theta\right)$$

$$E = \frac{e^{i(kR-\omega t)}}{R}\int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$

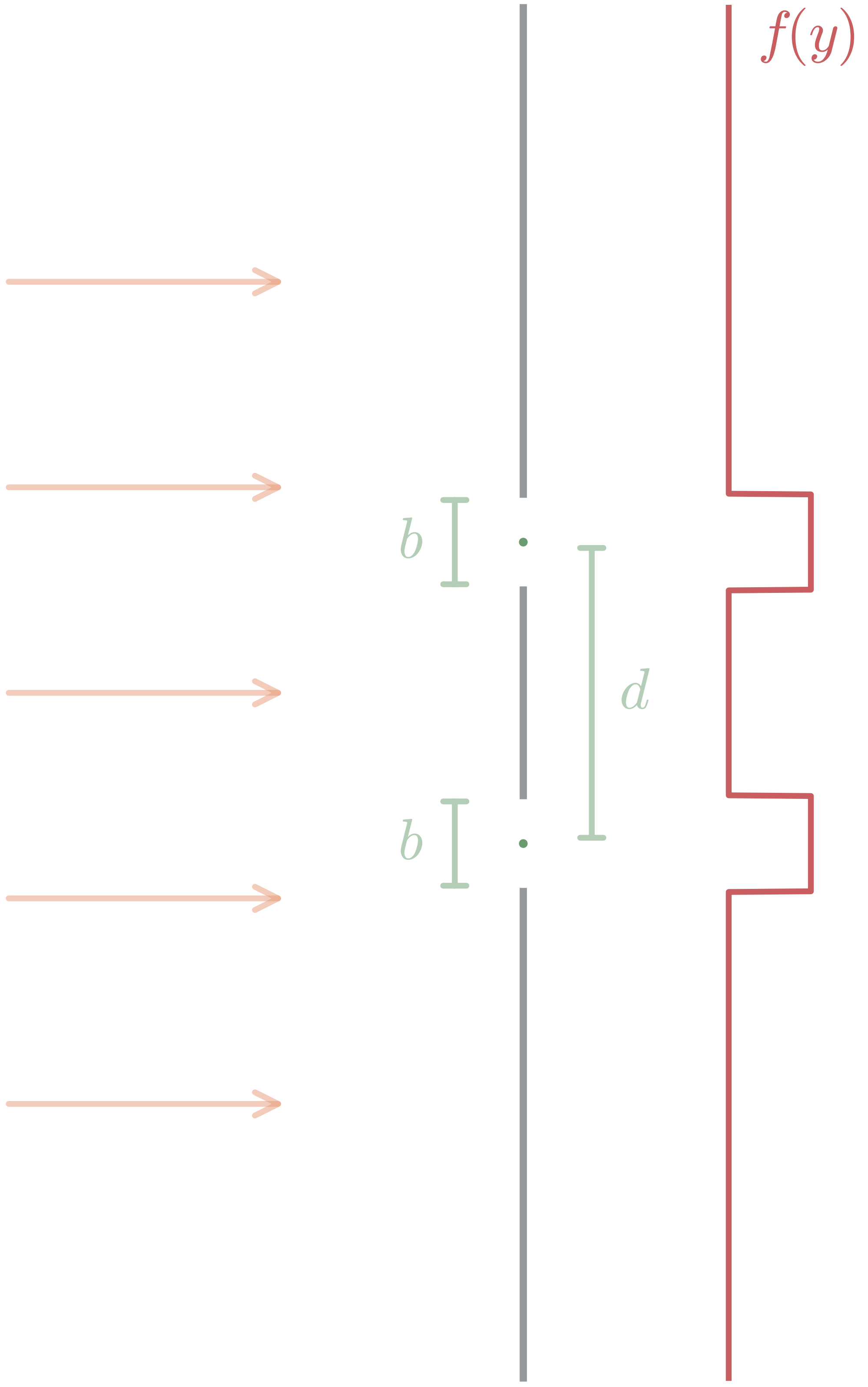


b
d
b

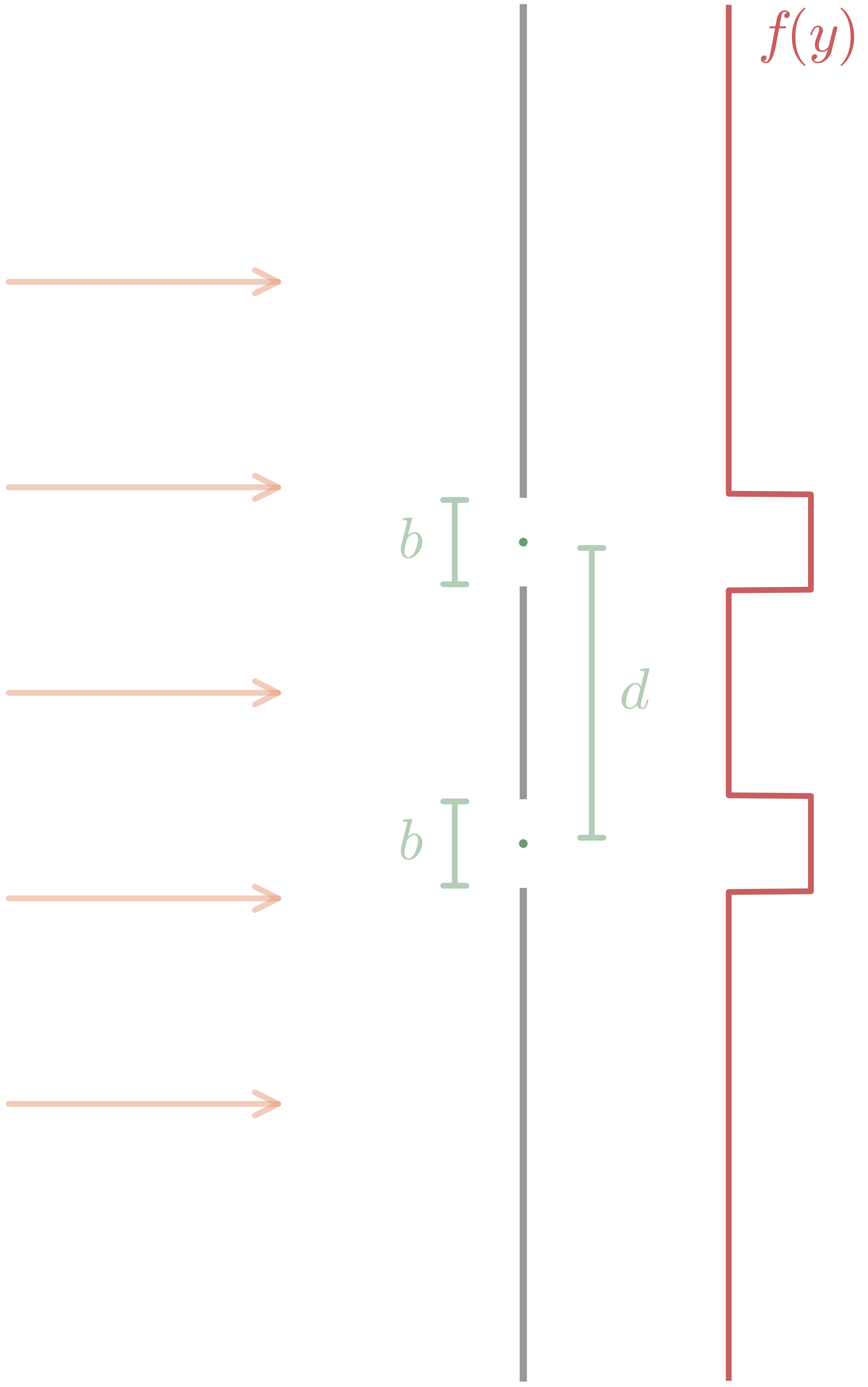

$\hat{y}$
$\frac{d}{2} + \frac{b}{2}$
$\frac{d}{2} - \frac{b}{2}$
0
$-\frac{d}{2} + \frac{b}{2}$
$-\frac{d}{2} - \frac{b}{2}$
b
d
b



$$f(y)=\begin{cases} E_0 & \text{si} \quad -\frac{d}{2}-\frac{b}{2}<y<-\frac{d}{2}+\frac{b}{2} \\ E_0 & \text{si} \quad \frac{d}{2}-\frac{b}{2}<y<\frac{d}{2}+\frac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$

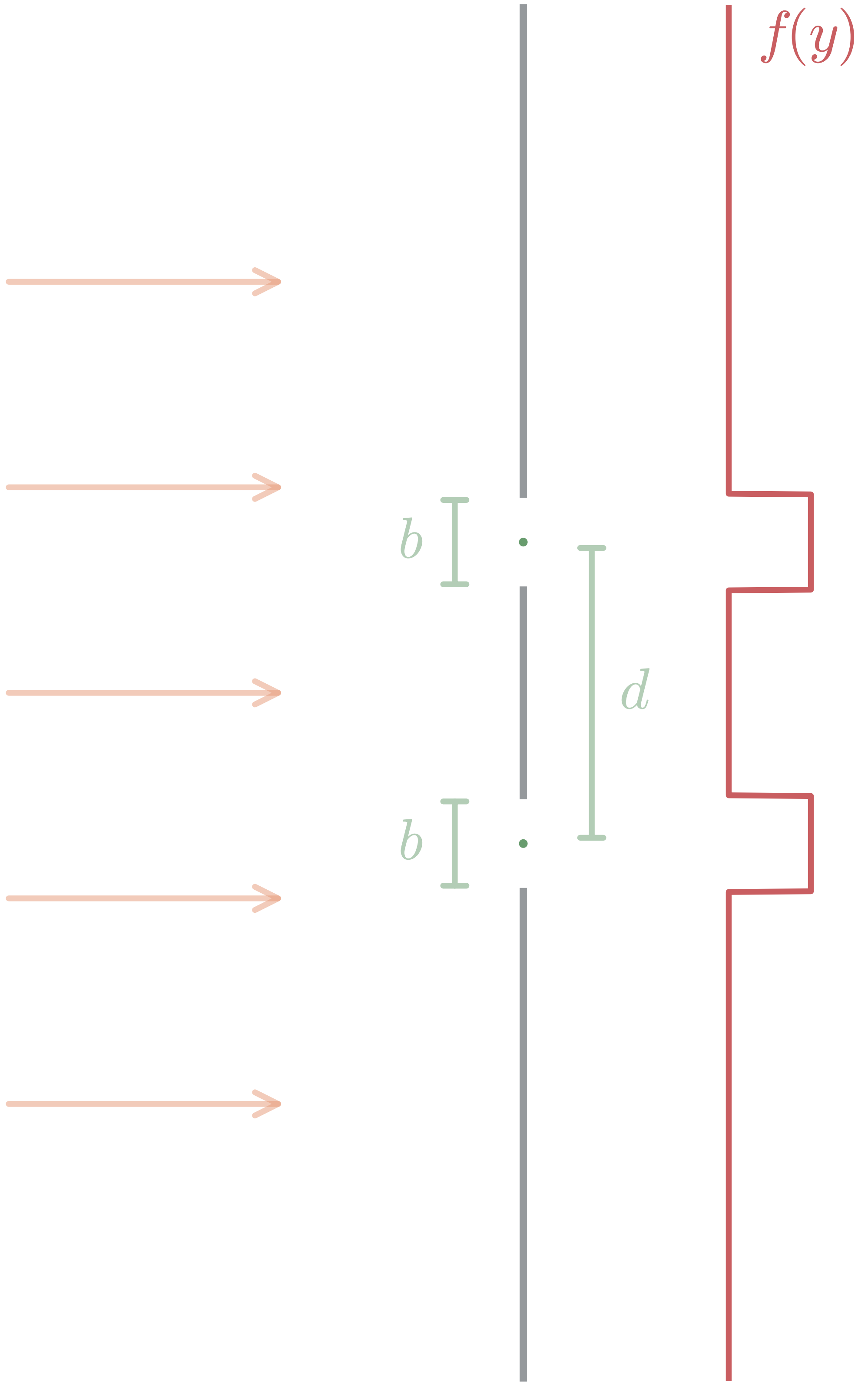


$$f(y) = \begin{cases} E_0 & \text{si} \quad -\dfrac{d}{2} - \dfrac{b}{2} < y < -\dfrac{d}{2} + \dfrac{b}{2} \\ E_0 & \text{si} \quad \dfrac{d}{2} - \dfrac{b}{2} < y < \dfrac{d}{2} + \dfrac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$



$$f(y) = \begin{cases} E_0 & \text{si} \quad -\frac{d}{2} - \frac{b}{2} < y < -\frac{d}{2} + \frac{b}{2} \\ E_0 & \text{si} \quad \frac{d}{2} - \frac{b}{2} < y < \frac{d}{2} + \frac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$

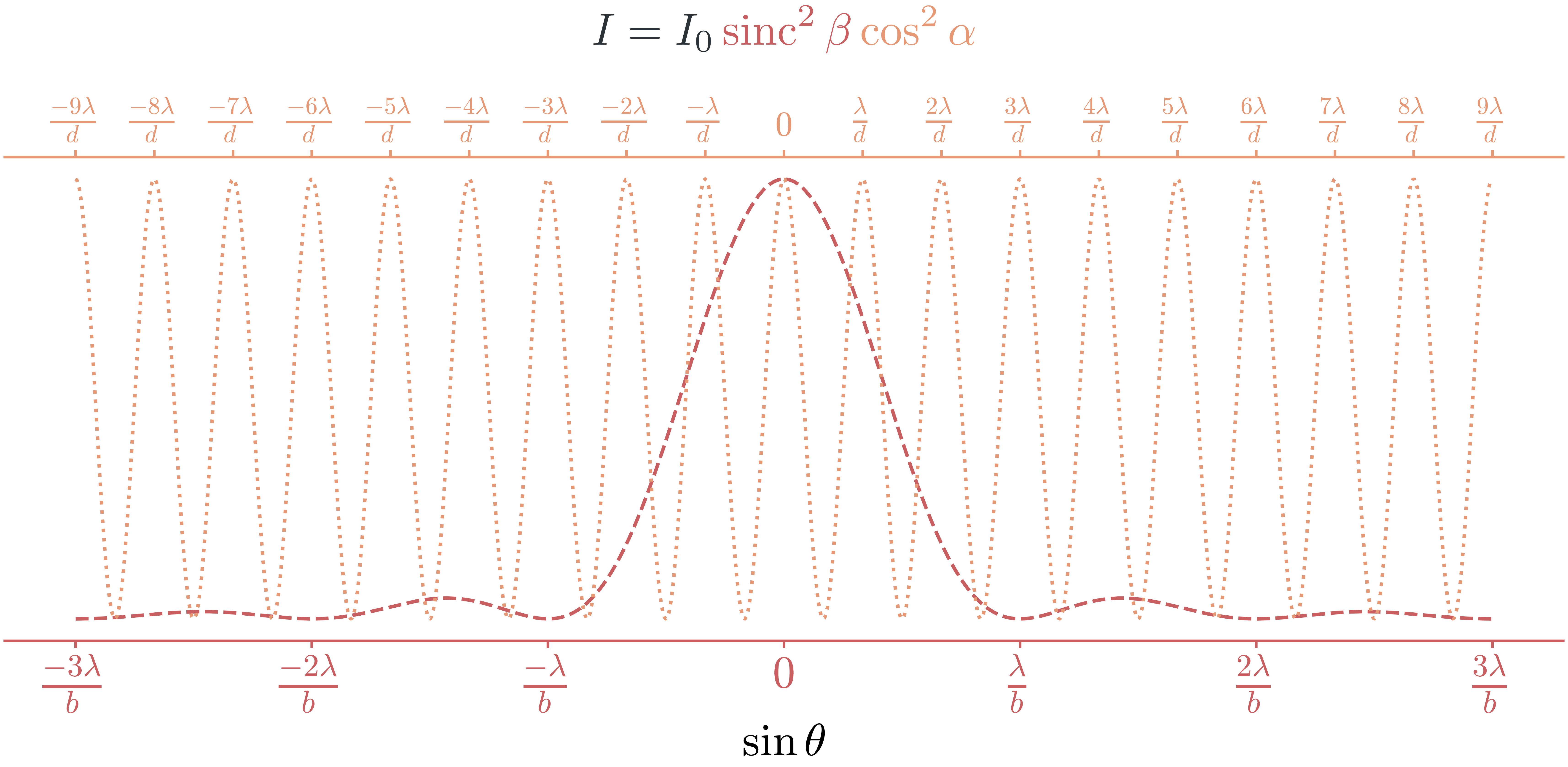
$$E = \frac{e^{i(kR-\omega t)}}{R} \int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$



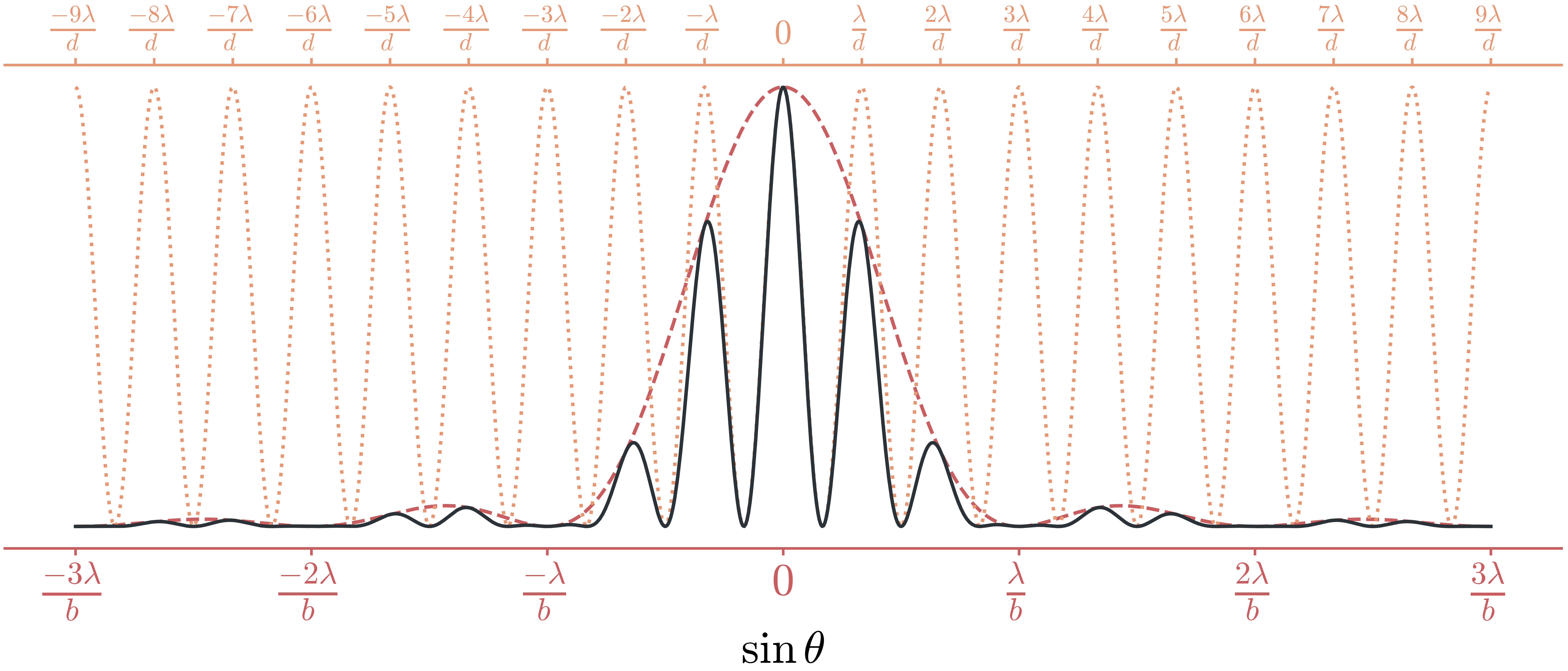
$$f(y) = \begin{cases} E_0 & \text{si} \quad -\frac{d}{2} - \frac{b}{2} < y < -\frac{d}{2} + \frac{b}{2} \\ E_0 & \text{si} \quad \frac{d}{2} - \frac{b}{2} < y < \frac{d}{2} + \frac{b}{2} \\ 0 & \text{en otro caso} \end{cases}$$

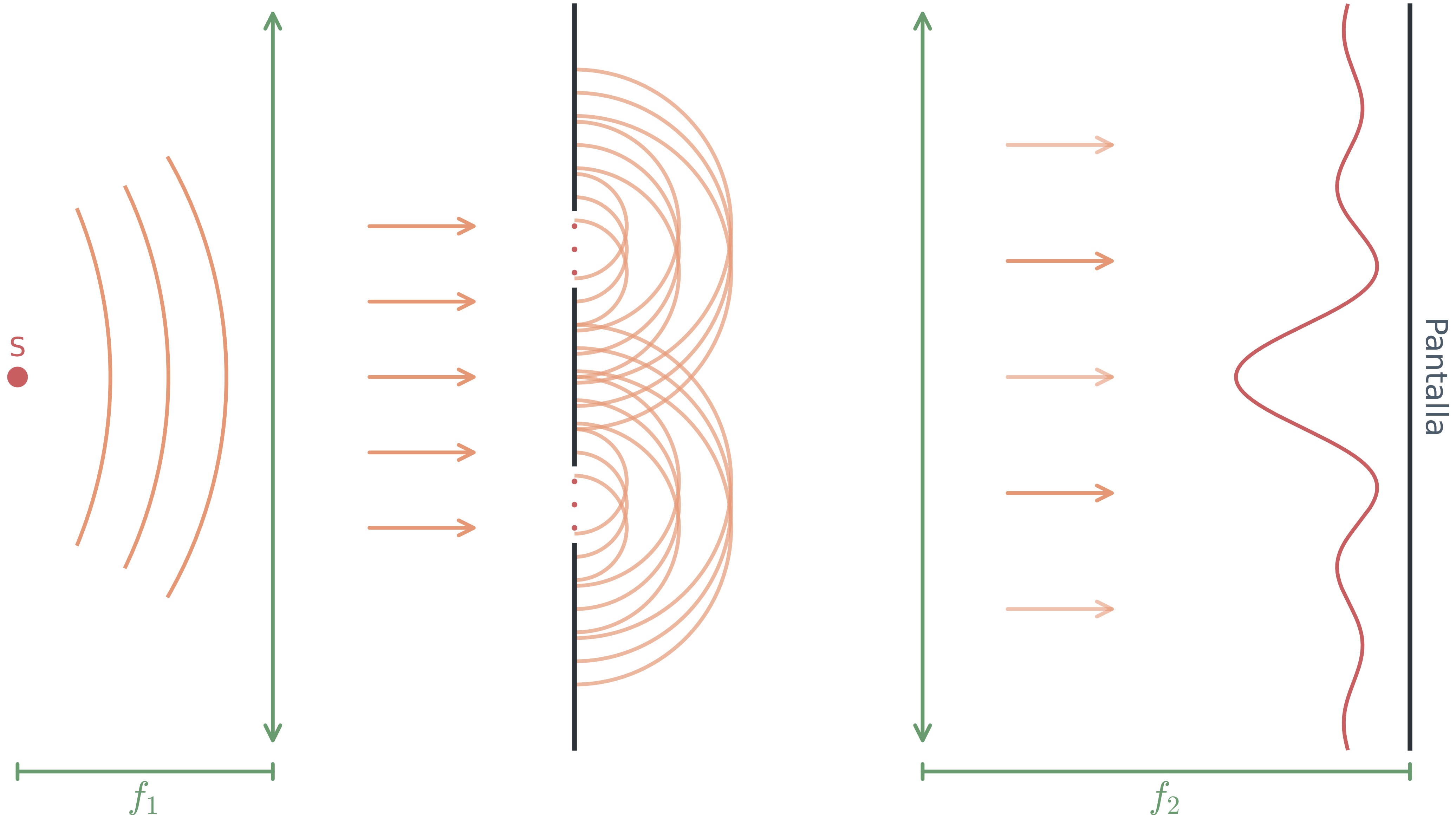
$$E = \frac{e^{i(kR-\omega t)}}{R} \int_{-\infty}^{\infty} f(y)\, e^{-iky\sin\theta}\, \mathrm{d}y$$

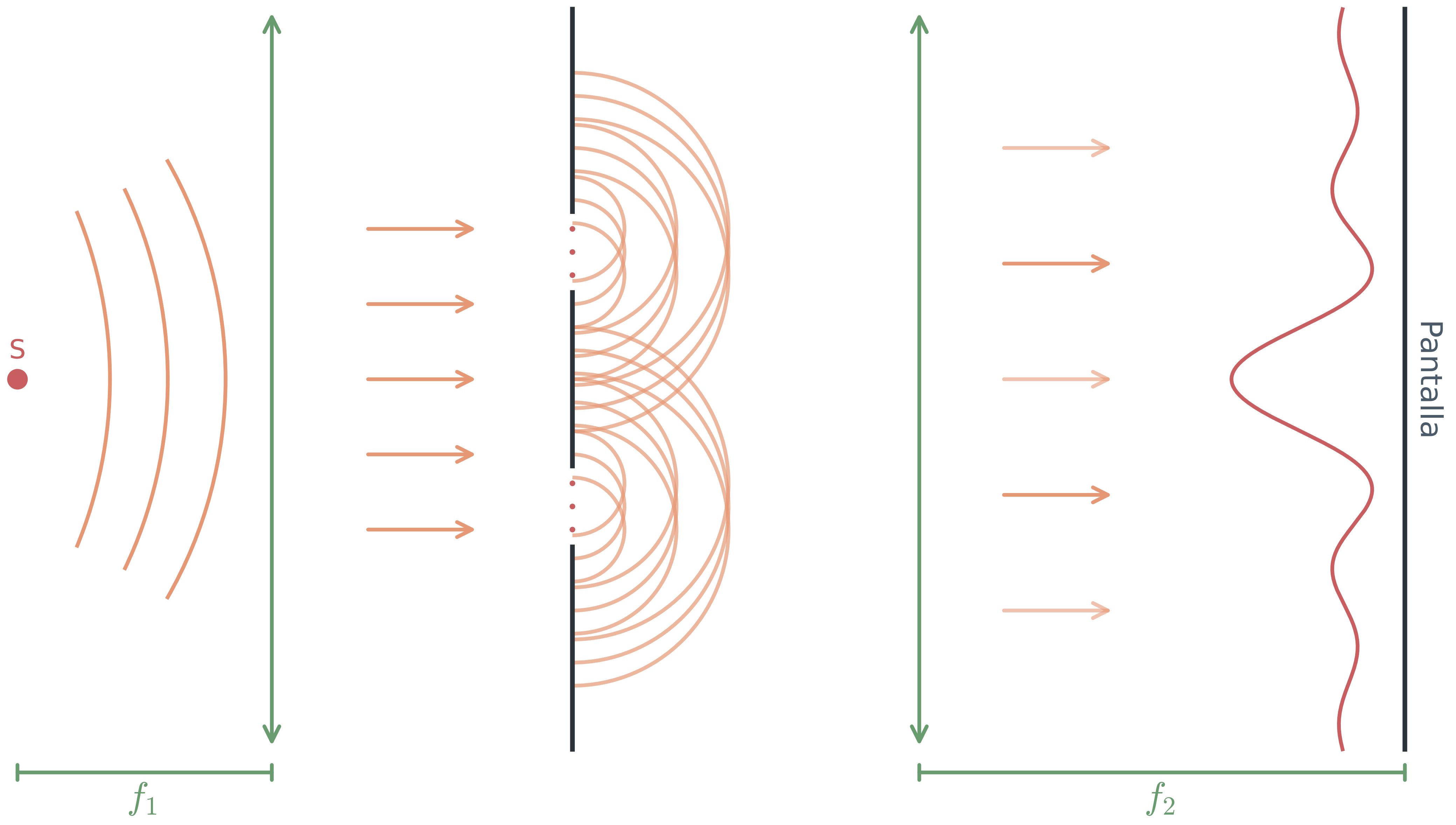
$$= \frac{e^{i(kR-\omega t)}}{R} \left[\int_{\frac{d}{2}-\frac{b}{2}}^{\frac{d}{2}+\frac{b}{2}} E_0\, e^{-iky\sin\theta}\, \mathrm{d}y + \int_{-\frac{d}{2}-\frac{b}{2}}^{-\frac{d}{2}+\frac{b}{2}} E_0\, e^{-iky\sin\theta}\, \mathrm{d}y \right]$$


$I = I_0 \operatorname{sinc}^2 \beta \cos^2 \alpha$
$\frac{-9\lambda}{d}$ $\frac{-8\lambda}{d}$ $\frac{-7\lambda}{d}$ $\frac{-6\lambda}{d}$ $\frac{-5\lambda}{d}$ $\frac{-4\lambda}{d}$ $\frac{-3\lambda}{d}$ $\frac{-2\lambda}{d}$ $\frac{-\lambda}{d}$ 0 $\frac{\lambda}{d}$ $\frac{2\lambda}{d}$ $\frac{3\lambda}{d}$ $\frac{4\lambda}{d}$ $\frac{5\lambda}{d}$ $\frac{6\lambda}{d}$ $\frac{7\lambda}{d}$ $\frac{8\lambda}{d}$ $\frac{9\lambda}{d}$
$\frac{-3\lambda}{b}$ $\frac{-2\lambda}{b}$ $\frac{-\lambda}{b}$ 0 $\frac{\lambda}{b}$ $\frac{2\lambda}{b}$ $\frac{3\lambda}{b}$
$\sin\theta$

$$I = I_0 \,\mathrm{sinc}^2 \beta \cos^2 \alpha$$




S
f_1
f_2
Pantalla

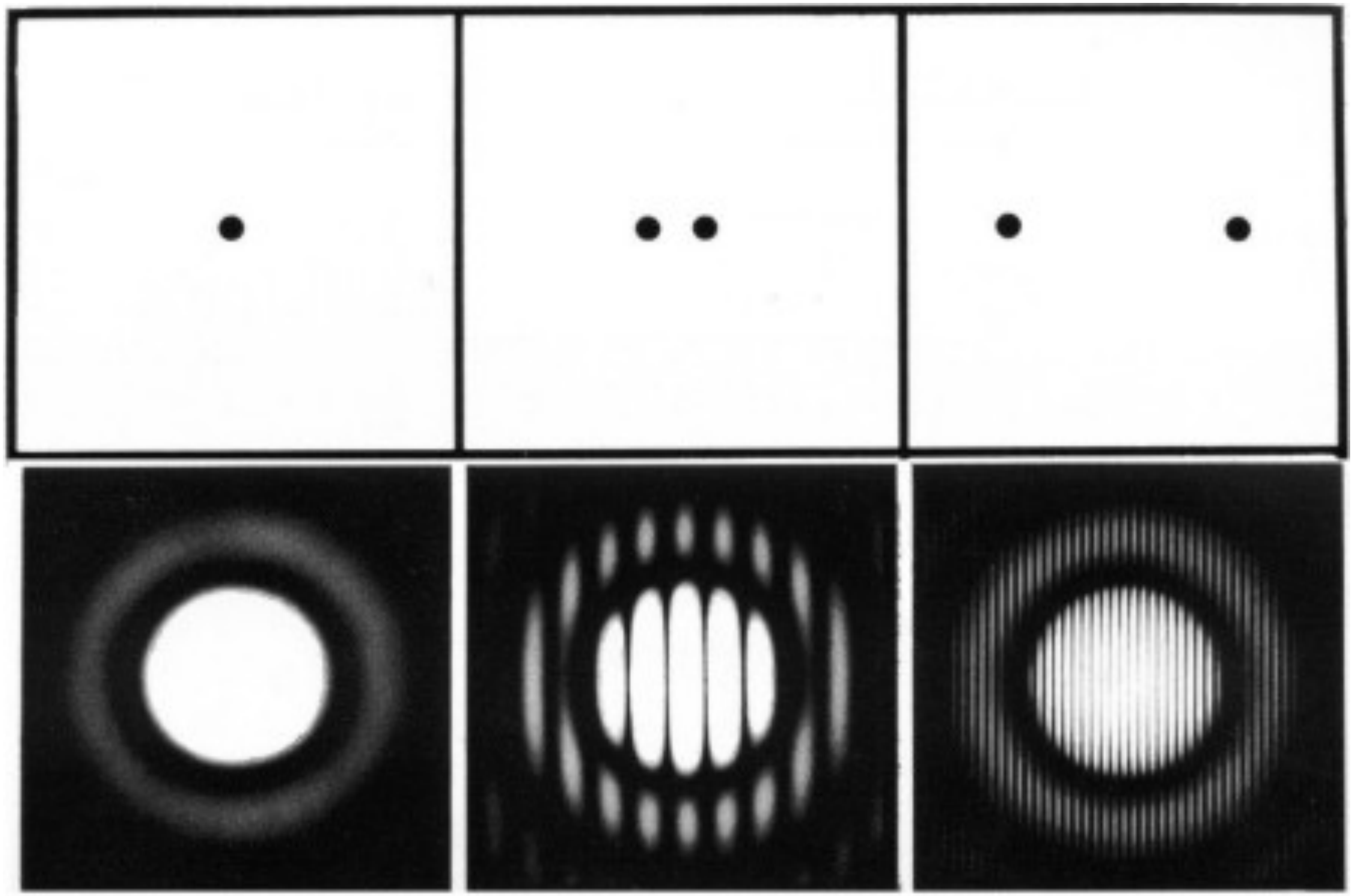


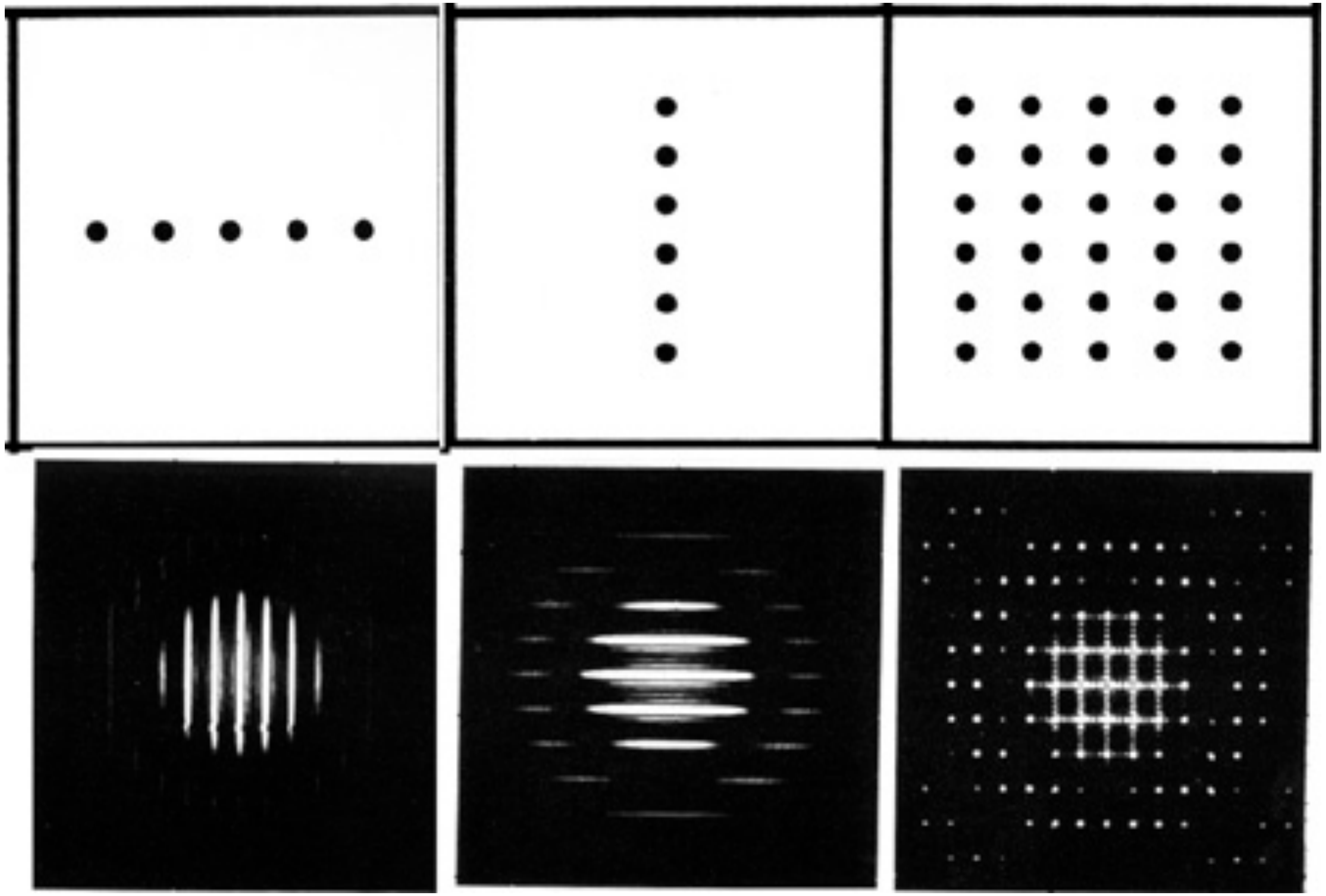
$$\sin\theta \simeq \theta \simeq \tan\theta \simeq y/f_2$$

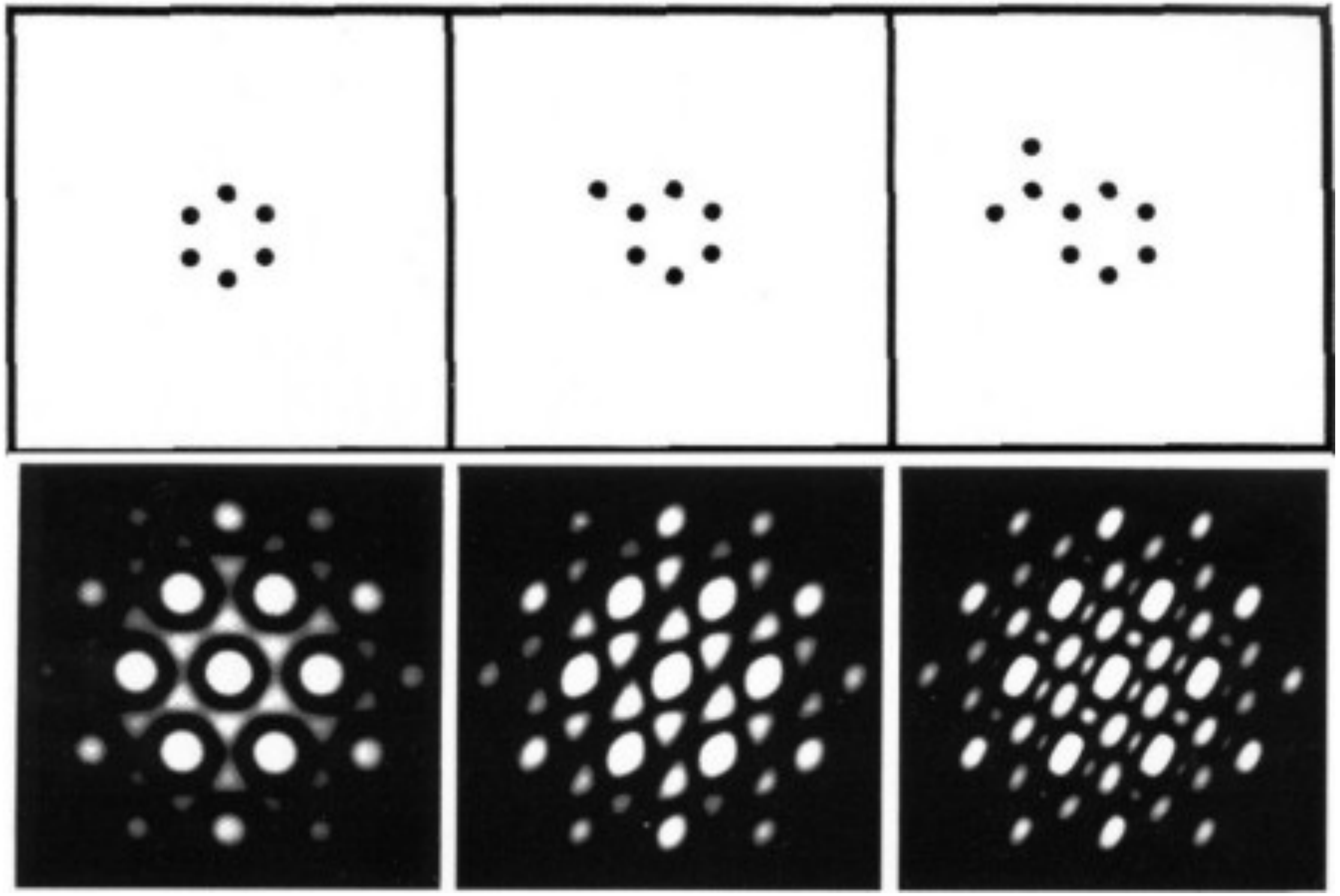
↑

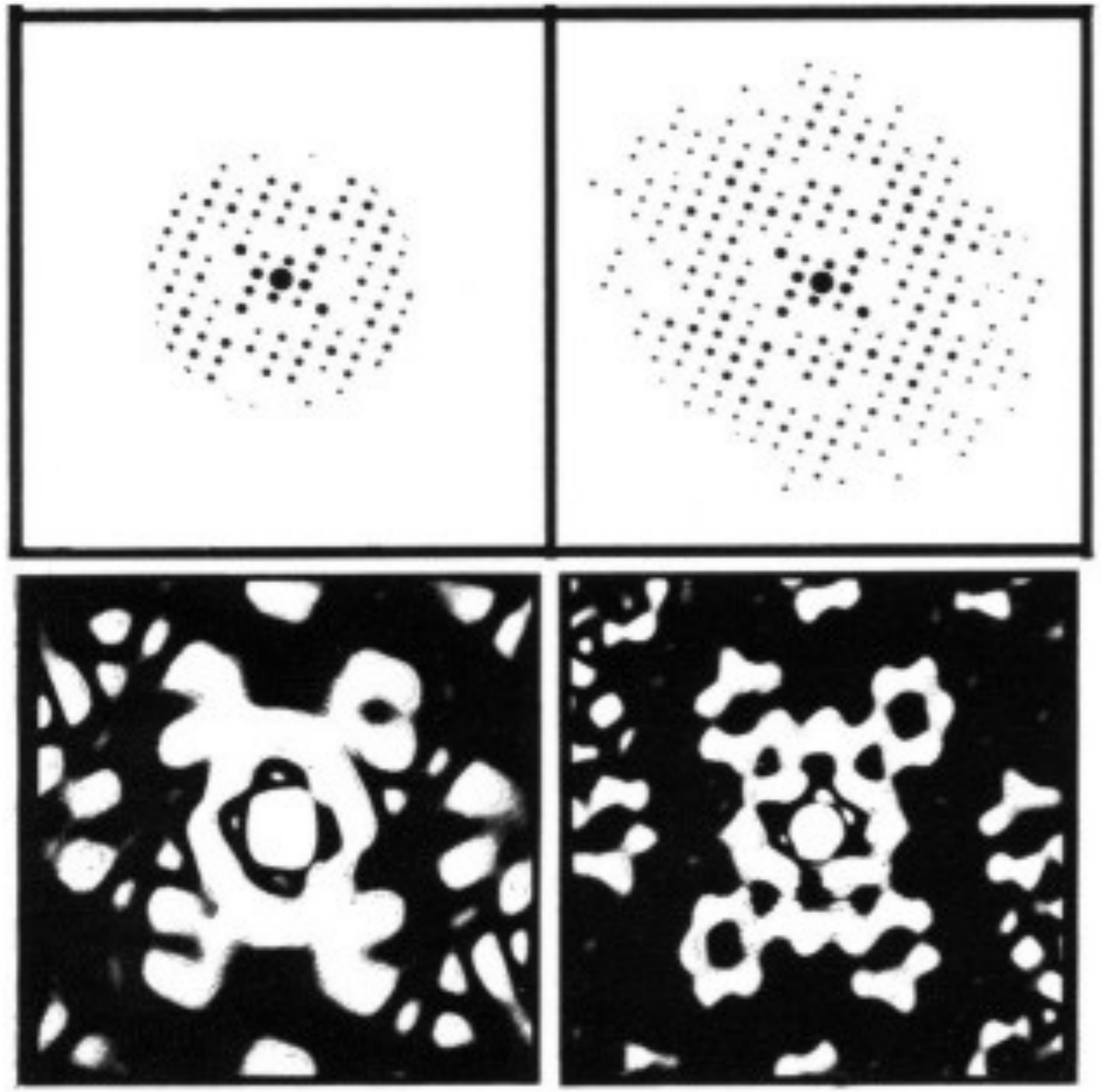
en este caso

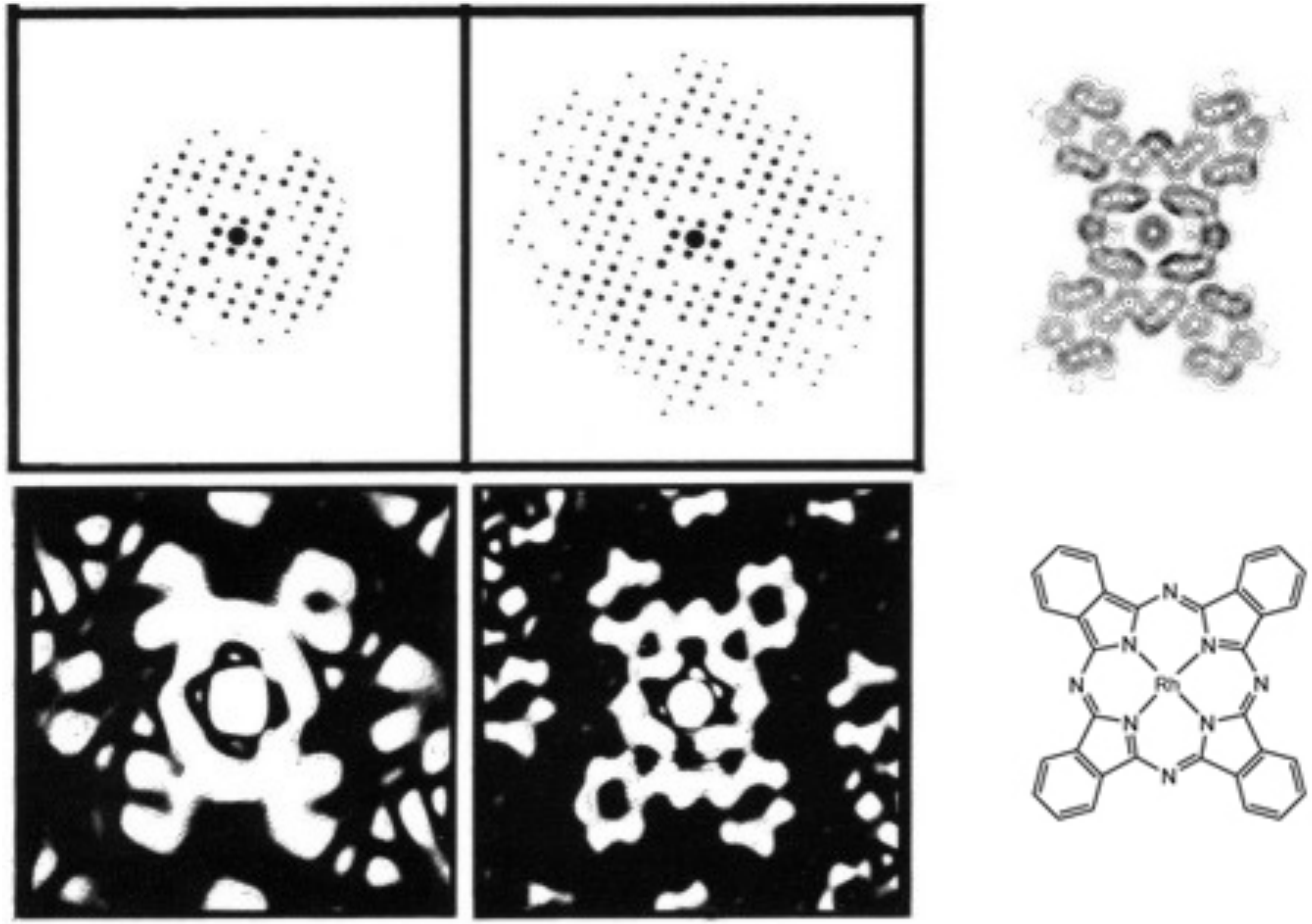
Algunos ejemplos en 2D

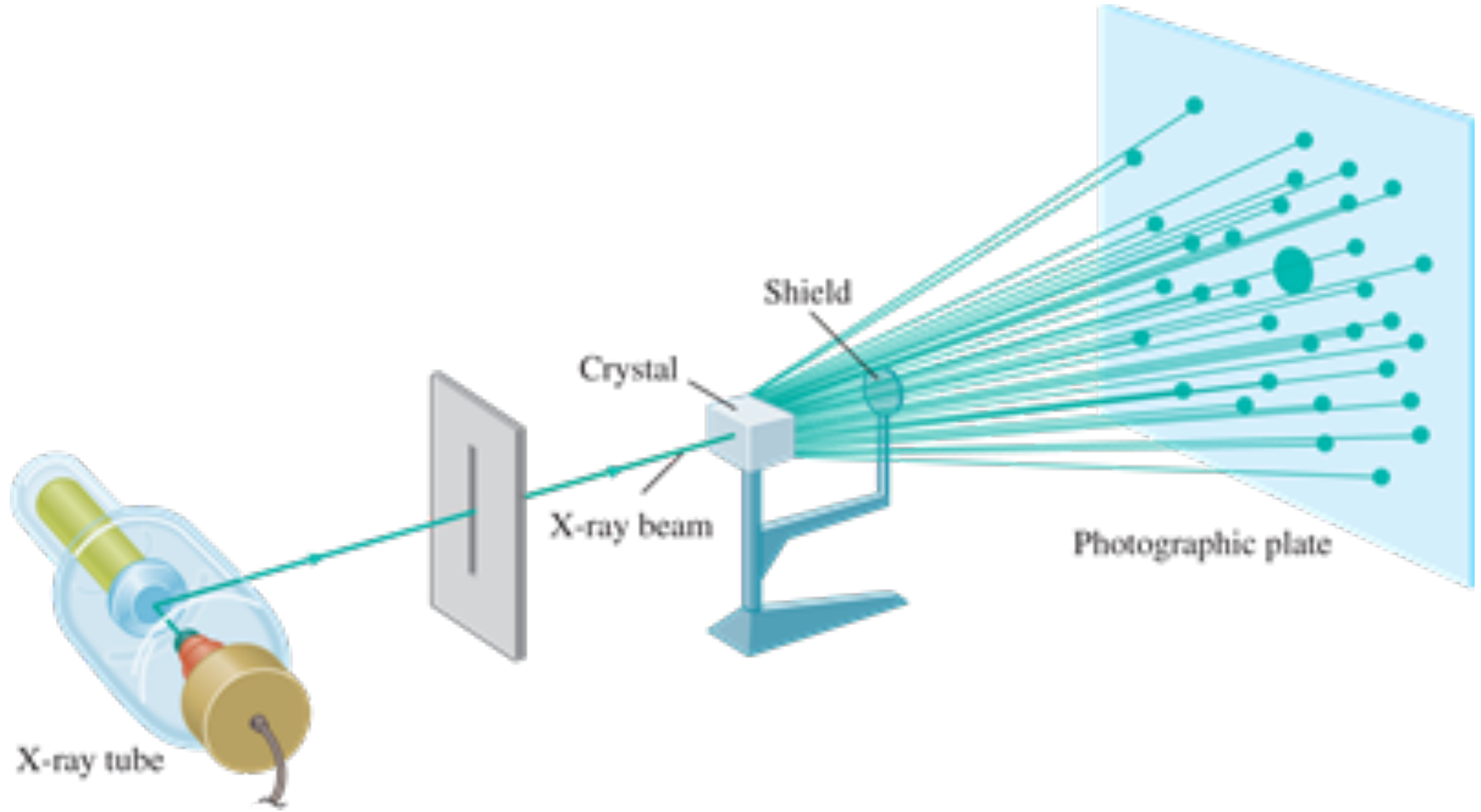









N
Rh


Shield
Crystal
X-ray beam
Photographic plate
X-ray tube

Esto es todo